

$x[n] = \mathcal{Z}^{-1}\{X(z)\}$	$X(z) = \mathcal{Z}\{x[n]\}$	
$a_1 x_1[n] + a_2 x_2[n]$	$a_1 X_1(z) + a_2 X_2(z)$	$X(z) = \sum_{n=-\infty}^{\infty} (a_1 x_1(n) + a_2 x_2(n)) z^{-n}$ $= a_1 \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + a_2 \sum_{n=-\infty}^{\infty} x_2(n) z^{-n}$ $= a_1 X_1(z) + a_2 X_2(z)$
$x_{Kr}[n] = \begin{cases} x[r], & n = Kr \\ 0, & n \notin K\mathbb{Z} \end{cases}$ with $K\mathbb{Z} := \{Kr : r \in \mathbb{Z}\}$	$X(z^K)$	$X_K(z) = \sum_{n=-\infty}^{\infty} x_K(n) z^{-n}$ $= \sum_{r=-\infty}^{\infty} x(r) z^{-rK}$ $= \sum_{r=-\infty}^{\infty} x(r) (z^K)^{-r}$ $= X(z^K)$
$x[Kn]$	$\frac{1}{K} \sum_{p=0}^{K-1} X\left(\frac{1}{z^K} \cdot e^{-i\frac{2\pi}{K}p}\right)$	ohio-state.edu or ee.ic.ac.uk
$x[n-k]$ with $k > 0$ and $x : x[n] = 0 \forall n < 0$	$z^{-k} X(z)$	$Z\{x[n-k]\} = \sum_{n=-\infty}^{\infty} x[n-k] z^{-n}$ $= \sum_{j=-k}^{\infty} x[j] z^{-(j+k)} \quad j = n - k$ $= \sum_{j=-k}^{\infty} x[j] z^{-j} z^{-k}$ $= z^{-k} \sum_{j=-k}^{\infty} x[j] z^{-j}$ $= z^{-k} \sum_{j=0}^{\infty} x[j] z^{-j} \quad x[\beta] = 0, \beta < 0$ $= z^{-k} X(z)$
$x[n+k]$ with $k > 0$	$z^k X(z) - z^k \sum_{n=0}^{k-1} x[n] z^{-n}$	
$x[n] - x[n-1]$ with $x[n]=0$ for $n < 0$	$(1 - z^{-1})X(z)$	
$x[n+1] - x[n]$	$(z-1)X(z) - zx[0]$	
$x[-n]$	$X(z^{-1})$	$Z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$ $= \sum_{m=-\infty}^{\infty} x(m) z^m$ $= \sum_{m=-\infty}^{\infty} x(m) (z^{-1})^{-m}$ $= X(z^{-1})$
$a^n x[n]$	$X(a^{-1}z)$	$Z\{a^n x[n]\} = \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n}$ $= \sum_{n=-\infty}^{\infty} x(n) (a^{-1}z)^{-n}$ $= X(a^{-1}z)$
$x^*[n]$	$X^*(z^*)$	$Z\{x^*(n)\} = \sum_{n=-\infty}^{\infty} x^*(n) z^{-n}$ $= \sum_{n=-\infty}^{\infty} [x(n)(z^*)^{-n}]^*$ $= \left[\sum_{n=-\infty}^{\infty} x(n) (z^*)^{-n} \right]^*$ $= X^*(z^*)$

$\text{Re}\{x[n]\}$	$\frac{1}{2} [X(z) + X^*(z^*)]$	
$\text{Im}\{x[n]\}$	$\frac{1}{2j} [X(z) - X^*(z^*)]$	
$nx[n]$	$-z \frac{dX(z)}{dz}$	$Z\{nx(n)\} = \sum_{n=-\infty}^{\infty} nx(n) z^{-n}$ $= z \sum_{n=-\infty}^{\infty} nx(n) z^{-n-1}$ $= -z \sum_{n=-\infty}^{\infty} x(n) (-nz^{-n-1})$ $= -z \sum_{n=-\infty}^{\infty} x(n) \frac{d}{dz} (z^{-n})$ $= -z \frac{dX(z)}{dz}$
$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$Z\{x_1(n) * x_2(n)\} = Z\left\{ \sum_{l=-\infty}^{\infty} x_1(l)x_2(n-l) \right\}$ $= \sum_{n=-\infty}^{\infty} \left[\sum_{l=-\infty}^{\infty} x_1(l)x_2(n-l) \right] z^{-n}$ $= \sum_{l=-\infty}^{\infty} x_1(l) \left[\sum_{n=-\infty}^{\infty} x_2(n-l) z^{-n} \right]$ $= \left[\sum_{l=-\infty}^{\infty} x_1(l) z^{-l} \right] \left[\sum_{n=-\infty}^{\infty} x_2(n) z^{-n} \right]$ $= X_1(z)X_2(z)$
$r_{x_1, x_2} = x_1^*[-n] * x_2[n]$	$R_{x_1, x_2}(z) = X_1^*\left(\frac{1}{z^*}\right)X_2(z)$	
$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1-z^{-1}} X(z)$	$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^n x[k] z^{-n} = \sum_{n=-\infty}^{\infty} (x[n] + \dots + x[-\infty]) z^{-n}$ $= X[z] (1 + z^{-1} + z^{-2} + \dots)$ $= X[z] \sum_{j=0}^{\infty} z^{-j}$ $= X[z] \frac{1}{1-z^{-1}}$
$x_1[n]x_2[n]$	$\frac{1}{2\pi} \oint_{\mathcal{C}} X_1(v)X_2\left(\frac{z}{v}\right)v^{-1}dv$	
$\sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n]$	$= \frac{1}{j2\pi} \oint_{\mathcal{C}} X_1(v)X_2^*\left(\frac{1}{v^*}\right)v^{-1}dv$	
$x[0] = \lim_{z \rightarrow \infty} X(z)$	$x[\infty] = \lim_{z \rightarrow 1} (z-1)X(z)$	