

Basics of Optical Telecommunications

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- Modeling of light
 - photons
 - electromagnetic waves
 - geometrical optics
 - field theory
- Optical networks
 - use
 - topologies
 - elements
- Standards

- Properties of optical fibers
 - geometry
 - modes
 - attenuation
 - dispersion
 - fabrication
- Nonlinear effects
 - Brillouin scattering
 - self-phase modulation
 - cross-phase modulation
 - four-wave mixing
 - Raman scattering

- Operation of lasers
 - Properties
 - applications
- Atomic energy levels
- Population inversion
- Energy bands in solid states
- Heterojunctions in semiconductors
- Quantum well lasers
- Vertical cavity surface emitting lasers
- Lasers as sources in optical telecommunications

- Amplifiers
 - Erbium doped fibers
 - Raman amplifiers
 - Semiconductor optical amplifiers
- Dispersion compensation
 - Dispersion shifted fibers
 - Dispersion compensating fiber
 - Compact dispersion compensation
- Detectors
 - PIN
 - APD

- Physical basics
 - electrooptic effect
 - magneto optic effect
 - acoustooptic effect
 - elasto optic effect
 - thermo optic effect
 - Bragg grating, Bragg mirrors
 - interferometers
- Modulation
- Switching

- Splitting
 - photolithography channels
 - fused fibers
 - interferometers
- Filtering, multiplexing, demultiplexing
 - prism
 - grating
 - Bragg layers
- WDM

- Nonlinear effects in fibers
- History of solitons
- Korteweg—deVries equations
- Envelop solitons
- Solitons in optical fibers
- Amplification of solitons – optical soliton transmission systems

Photon model

- particles with energy $h\nu$,
- bosons
- useful in
 - quantum mechanics
 - particle physics
 - telecommunications
electron excitations: lasers, detectors

Electromagnetic wave model

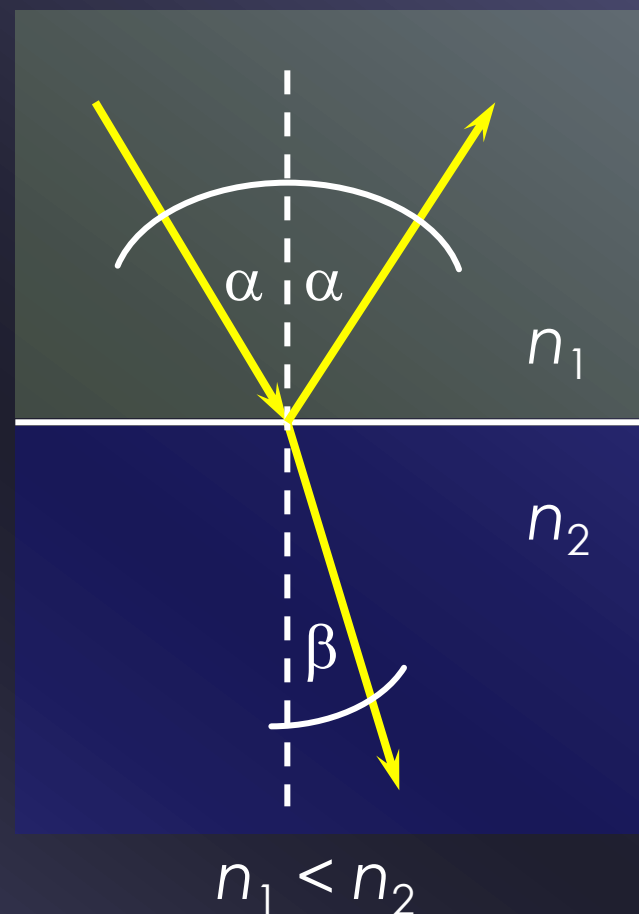
- the Maxwell equations describe the behavior
- $c = (\mu_0 \epsilon_0)^{-1/2}$ velocity of light in vacuum
- $v = (\mu \epsilon)^{-1/2}$ velocity of propagation in materials
- refraction index: $n = (\mu_r \epsilon_r)^{-1/2}$
- used in optical telecommunication
 - modeling the fiber as waveguide

Geometrical optics

- rays
- Snellius—Descartes law

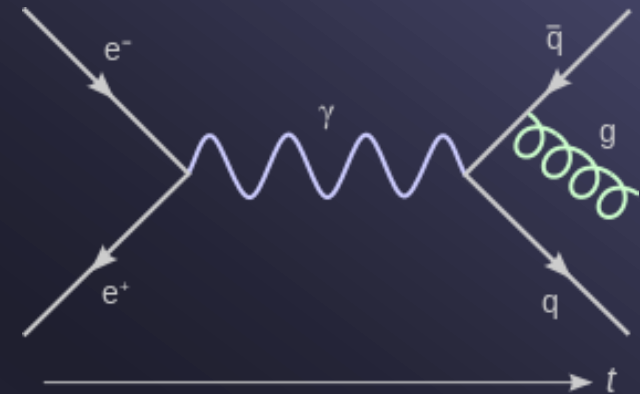
$$n_1 \sin \alpha = n_2 \sin \beta$$

- reflection and transmission



Field theory

- force carrier particle in electromagnetic interaction
- particle-like excitation – some say, it is a spread out field unit that generates and collapses
- similar to gluons, W, Z bosons, and the Higgs bosons – they are generated due to spontaneous symmetry breaking
- useful in
 - particle physics
 - telecommunications – not yet



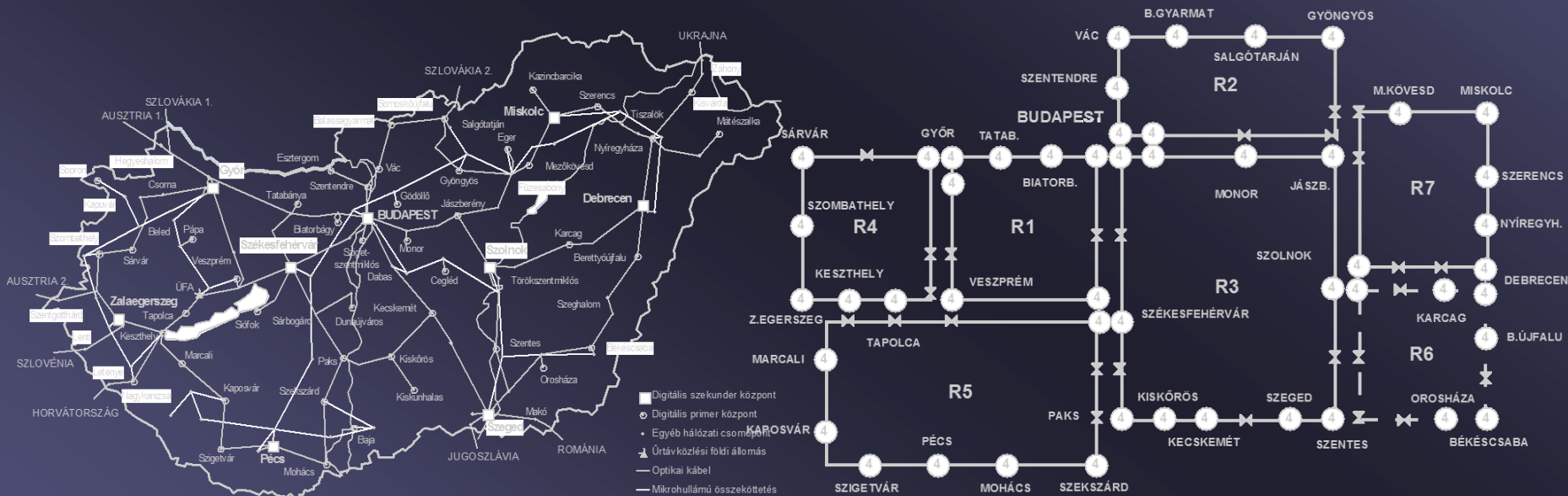
By Joel Holdsworth (Joelholdsworth) - Non-Derived SVG of Radiate_gluon.png, originally the work of SilverStar at Feynmann-diagram-gluon-radiation.svg, updated by joelholdsworth., CC BY 2.5, <https://commons.wikimedia.org/w/index.php?curid=1764161>

Levels

- Intercontinental – exclusive, silica cables
- long haul continental – almost exclusive, silica
- national backbones – almost exclusive, silica
- regional backbones – mostly optical, silica
- local – still lot of copper, but FTTX increasing, silica or plastic
- indoors – rare, mostly plastic optical cables
- intra vehicle – rare, plastic (glass is not suitable)

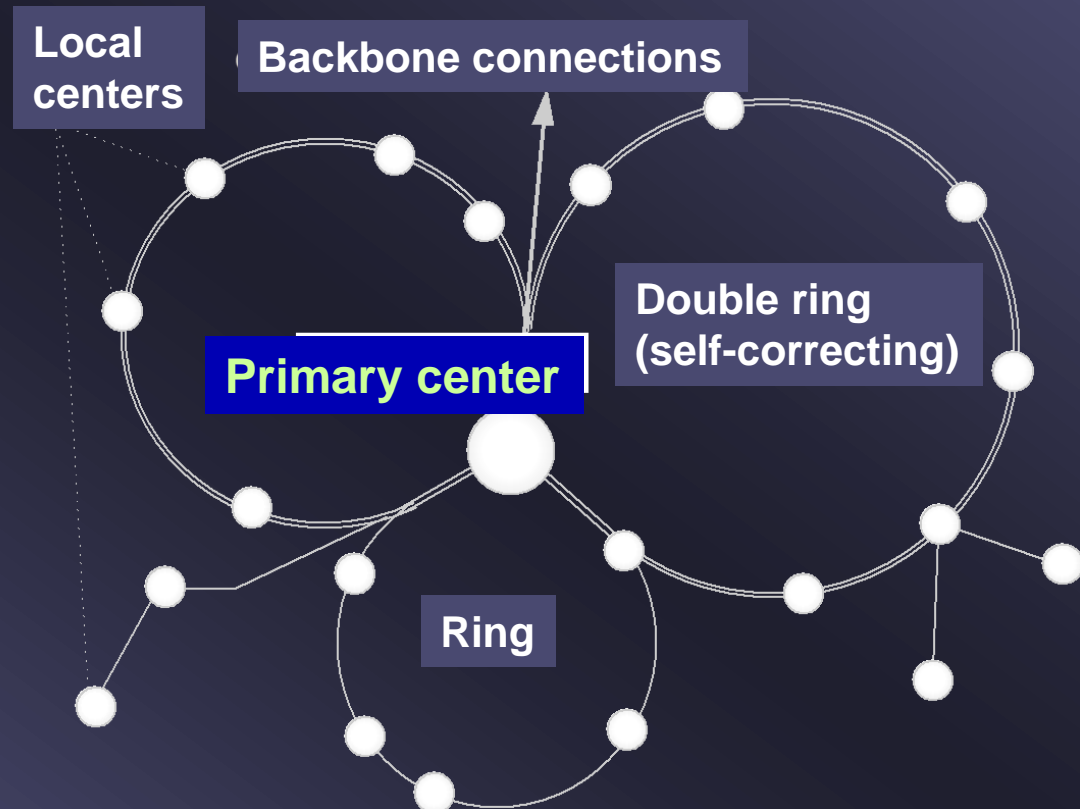
Topologies

- Intercontinental to regional backbones – reserved double ring, mesh



Topologies

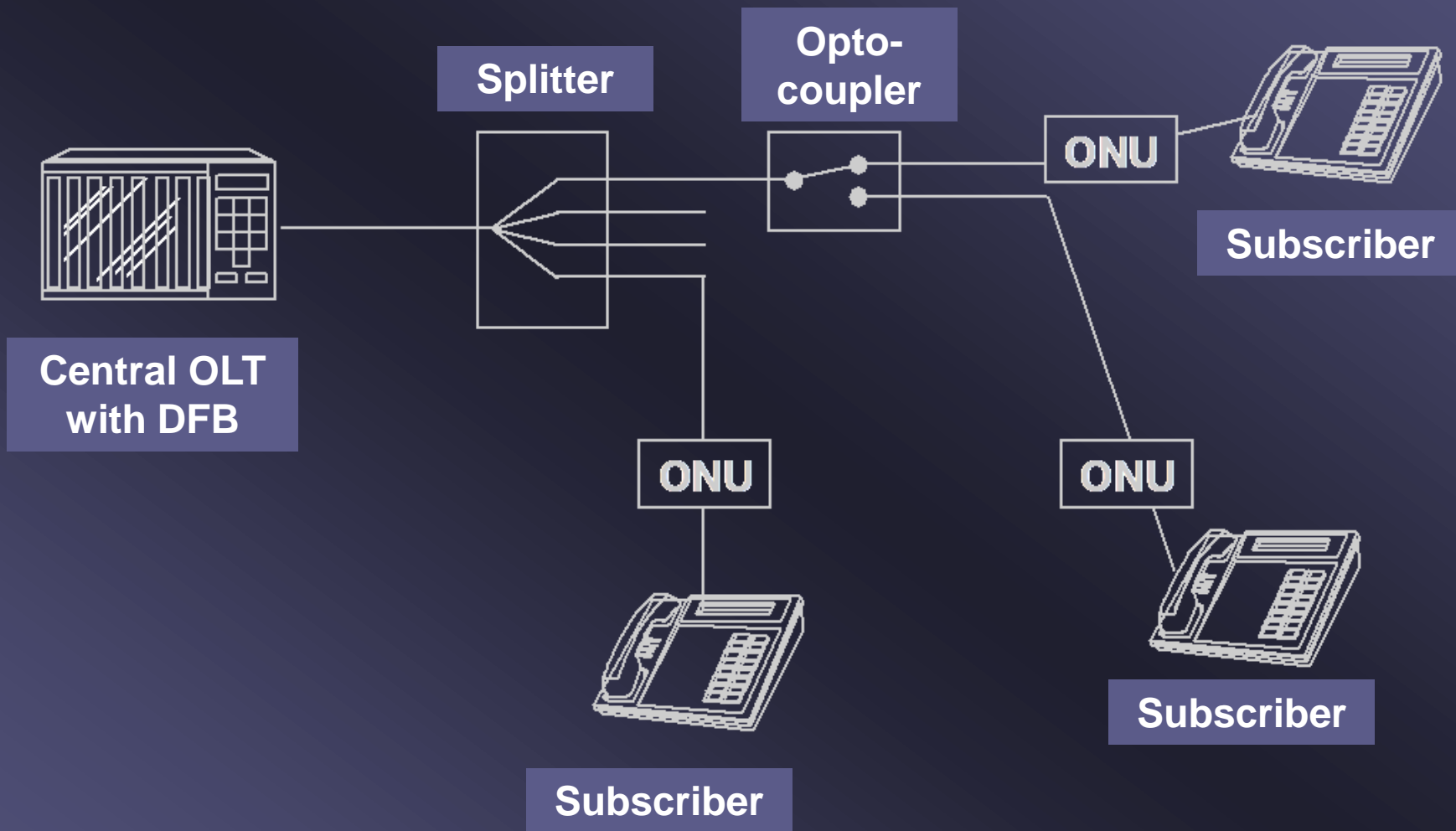
- Intercontinental to regional backbones – reserved double ring, mesh
- local – mostly tree



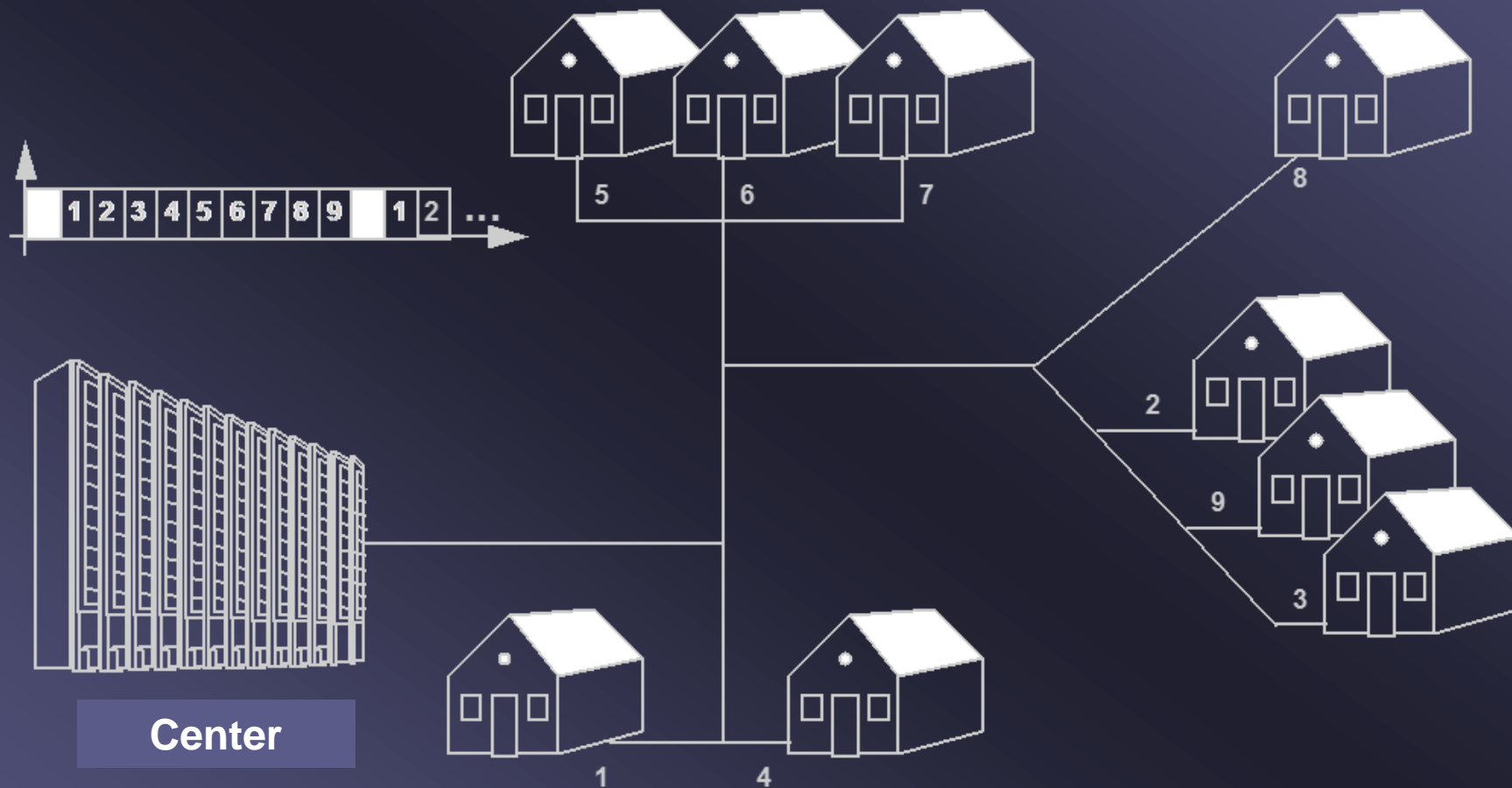
Topologies

- Intercontinental to regional backbones –double ring (reserved), mesh
- local – mostly tree (PON)
- indoors – mostly star
- intra vehicle – tree or bus

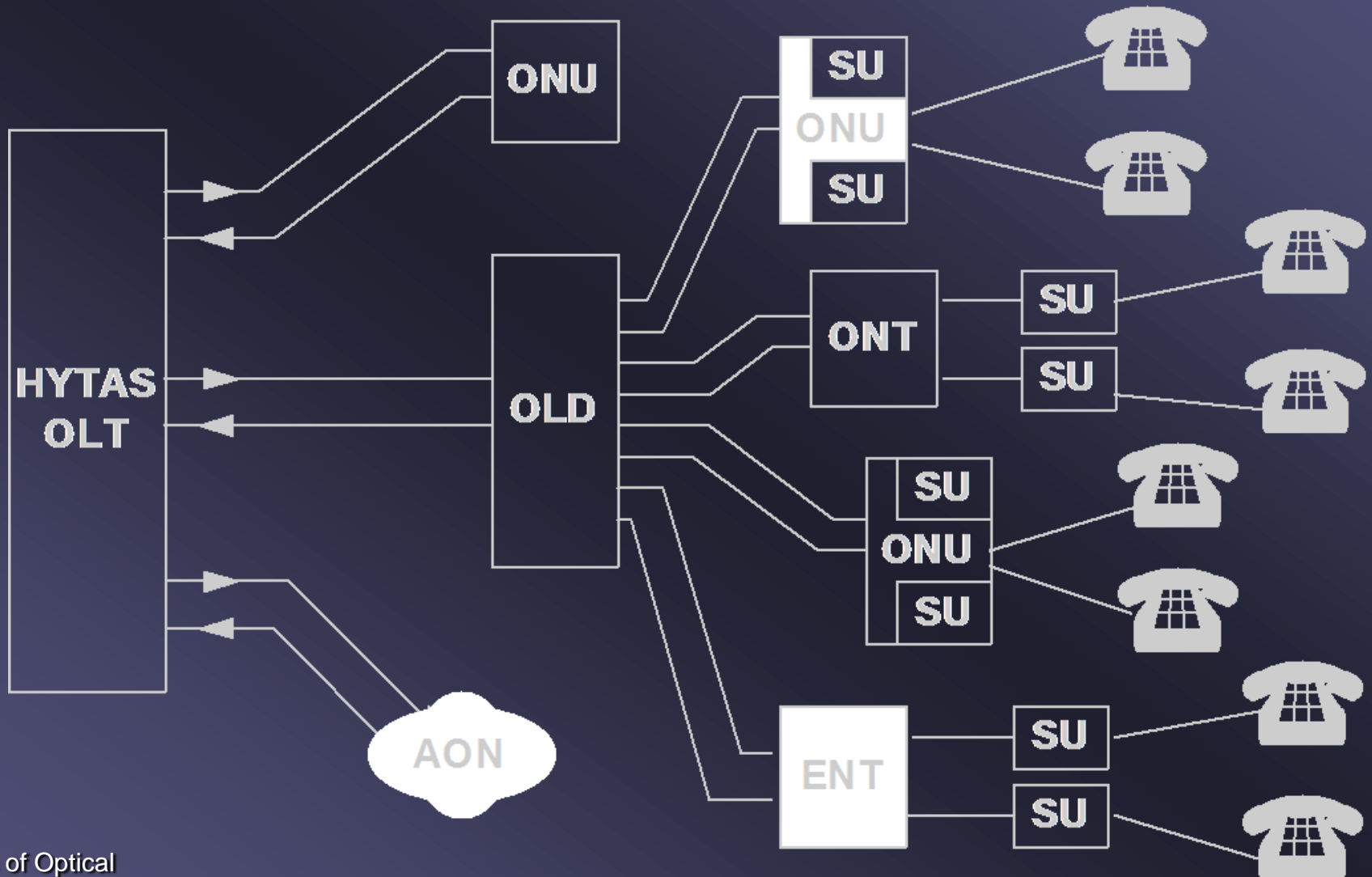
Passive optical networks (PON)



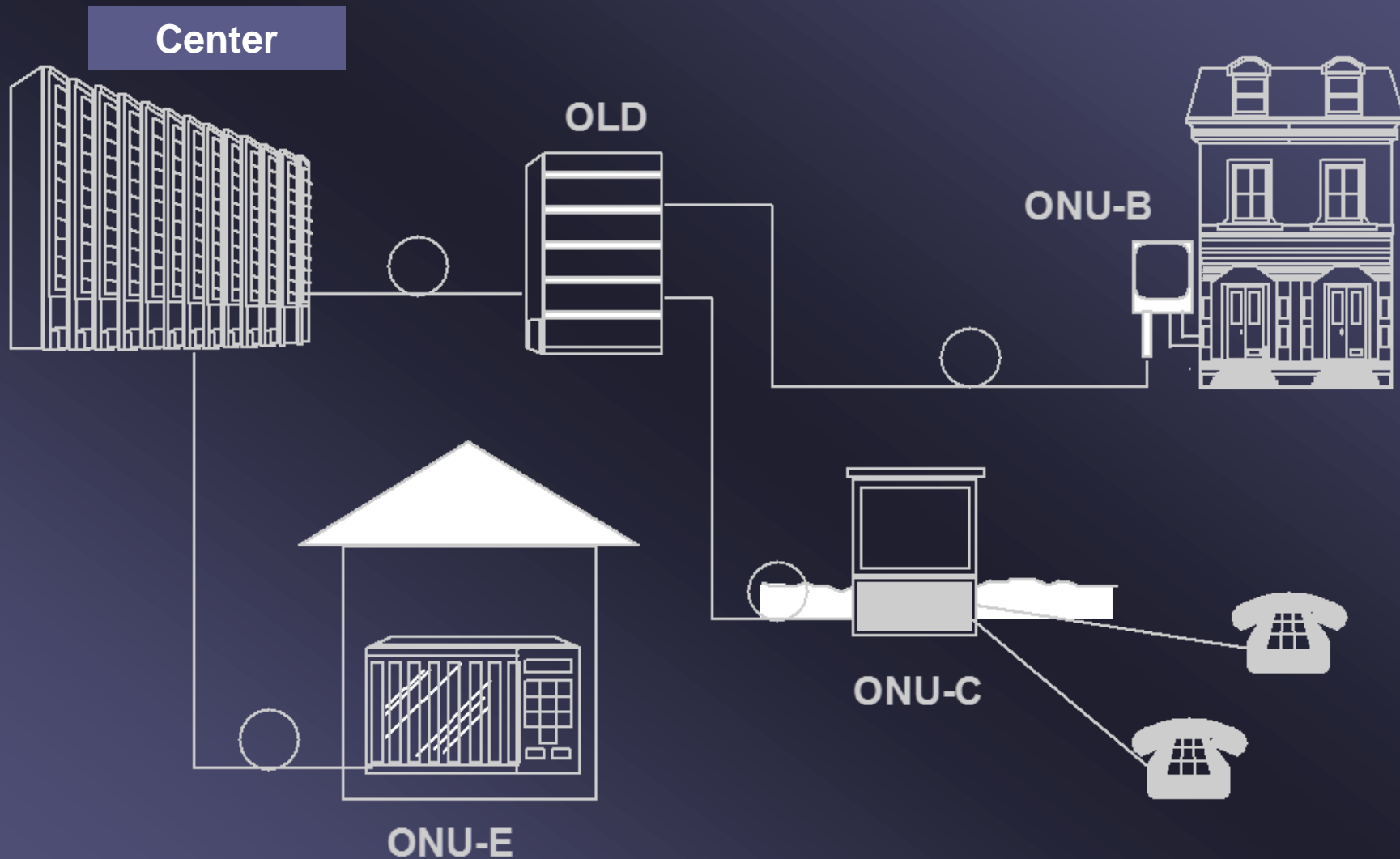
TDMA in PON



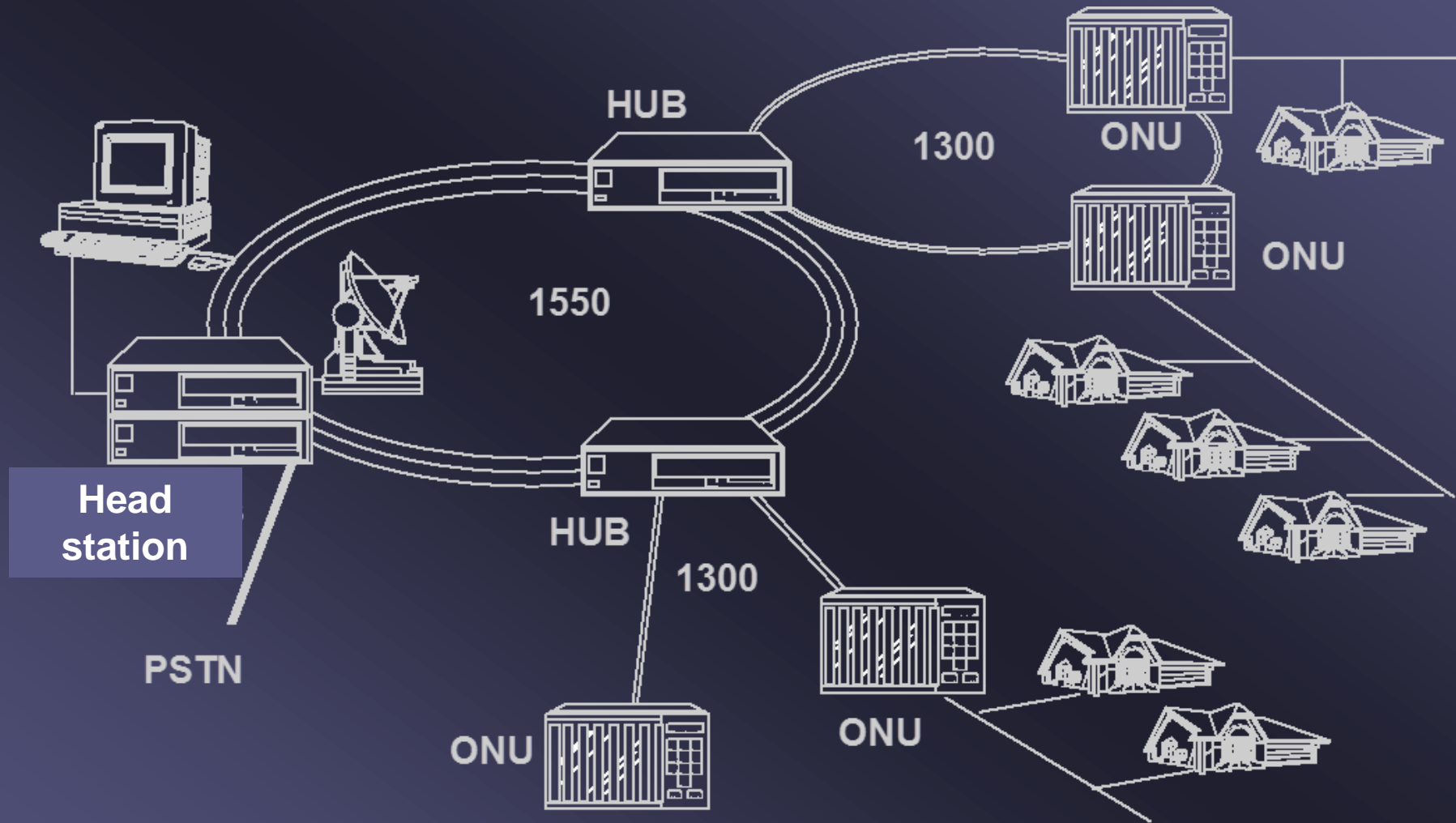
Hytas



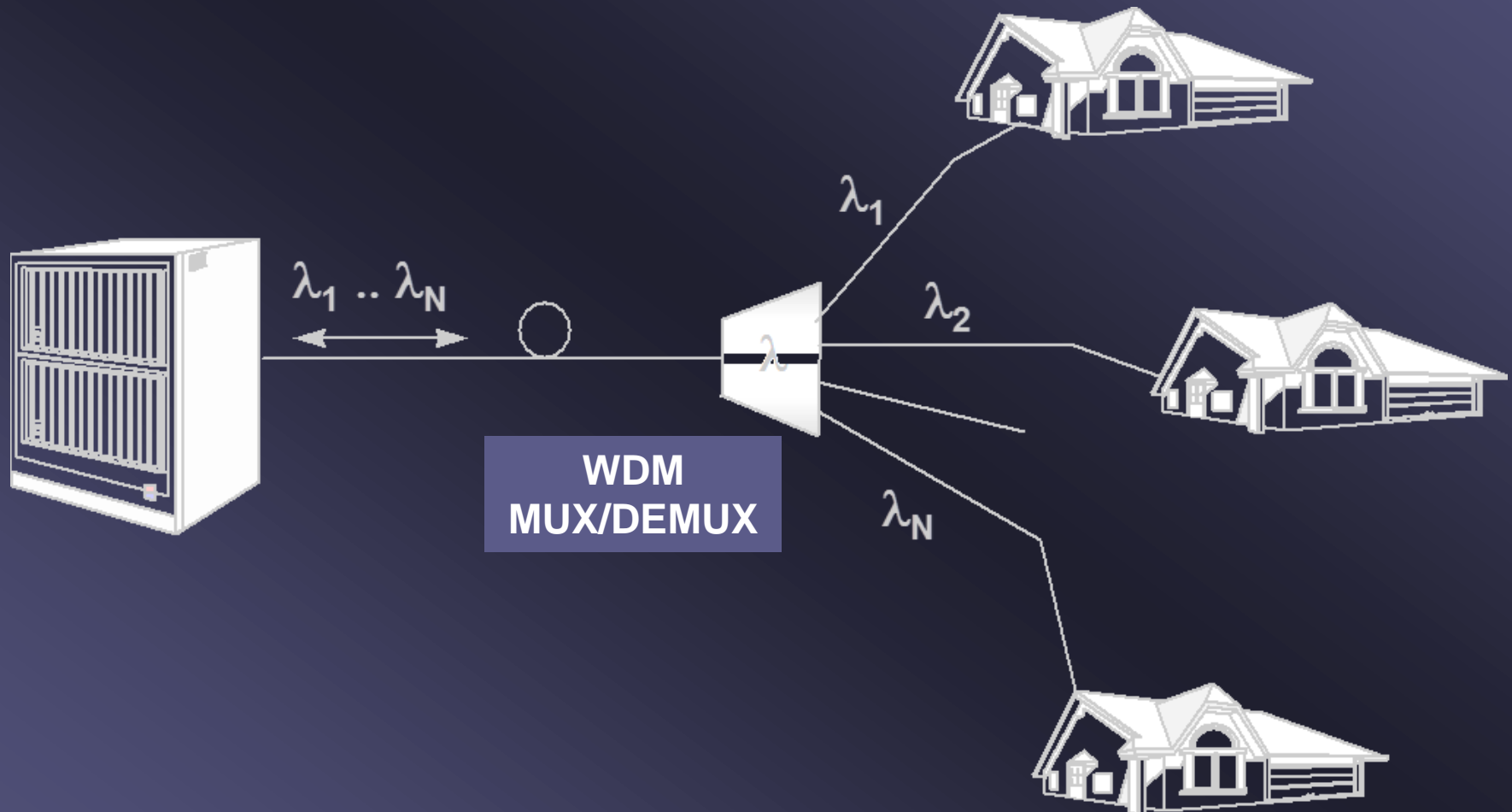
Subscriber units



Cable TV networks



WDM in PON access networks



Elements

● Passive

- fiber
- splitter
- multiplexer/demultiplexer
- dispersion compensator
- switch

● Active

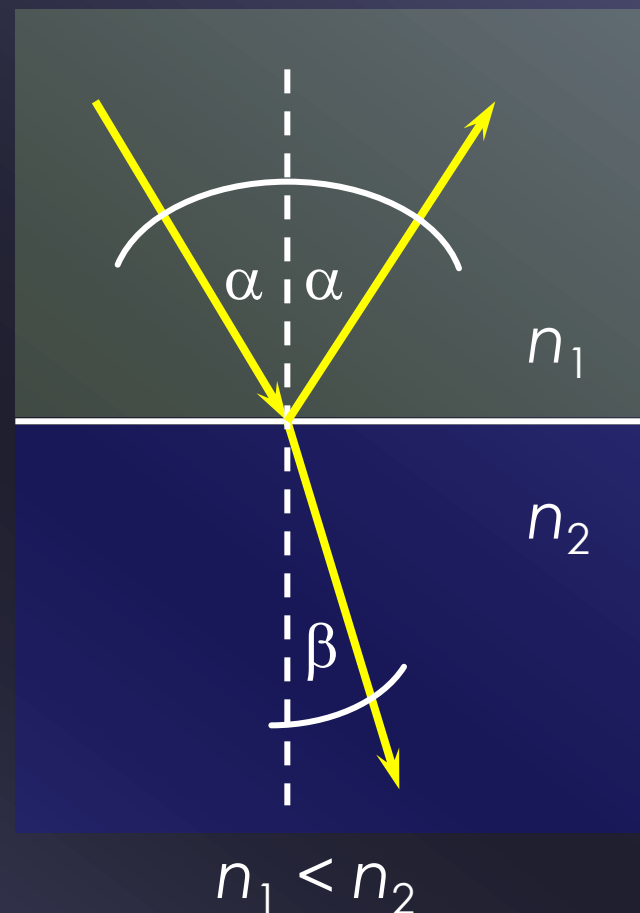
- source, detector
- amplifier
- modulator

Geometrical optics

- rays
- Snellius—Descartes law

$$n_1 \sin \alpha = n_2 \sin \beta$$

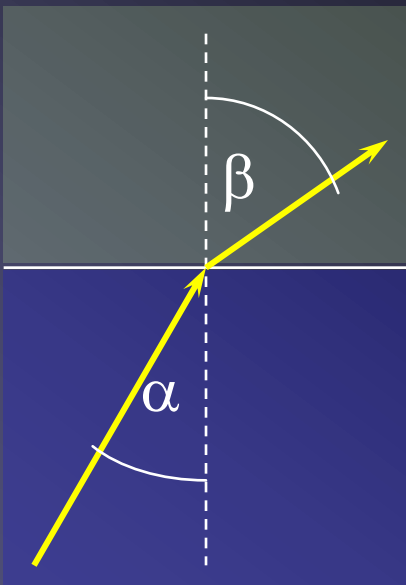
- reflection and transmission



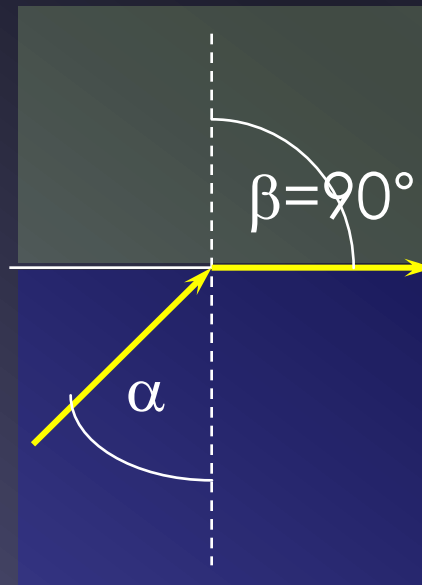
Geometrical optics

- total internal reflection – no transmitted light

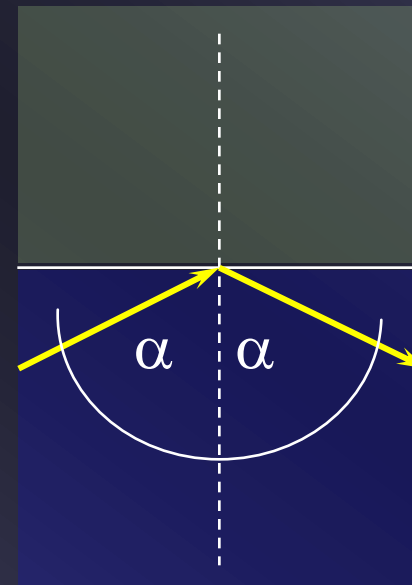
$$\alpha < \alpha_c$$



$$\alpha = \alpha_c$$

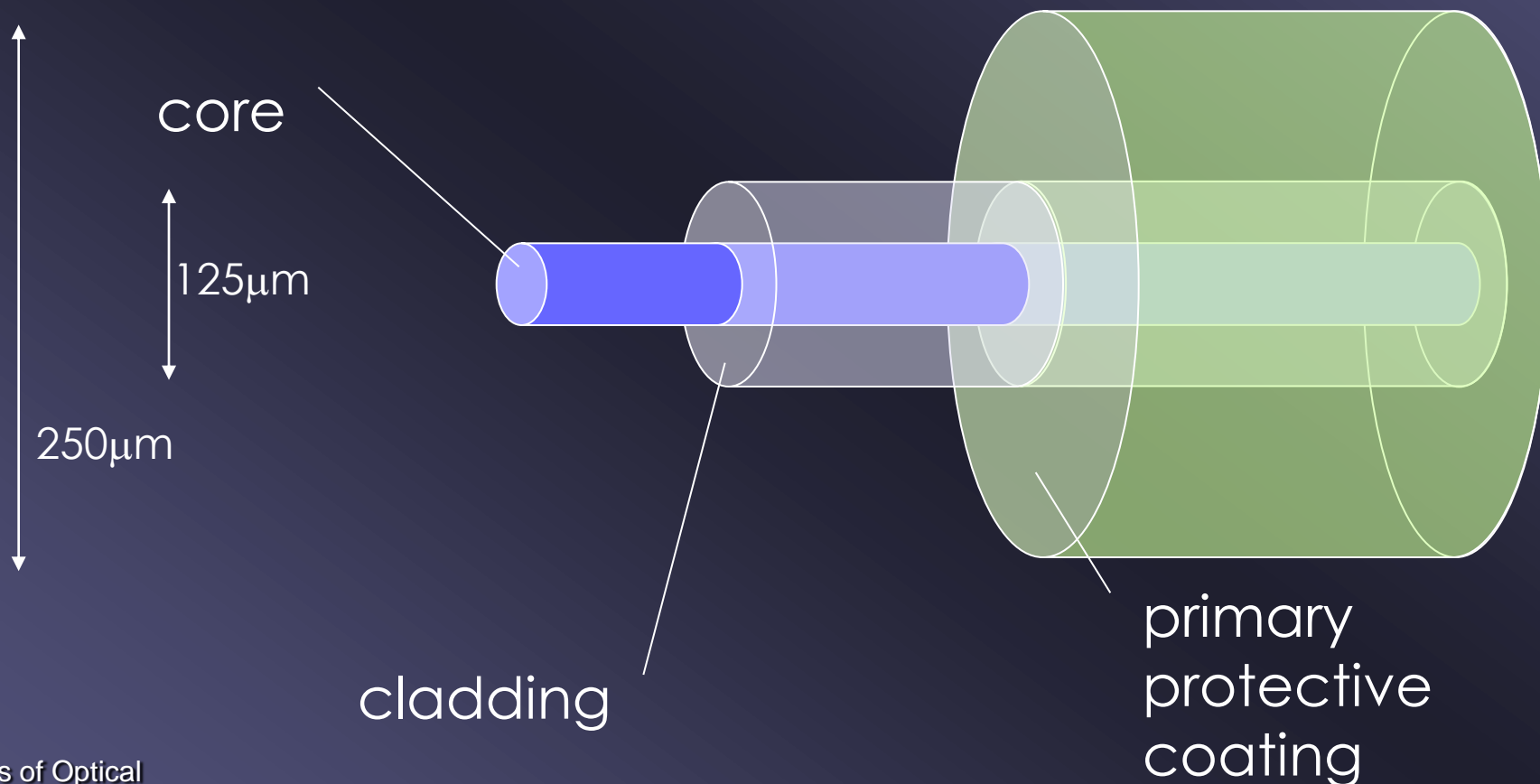


$$\alpha > \alpha_c$$

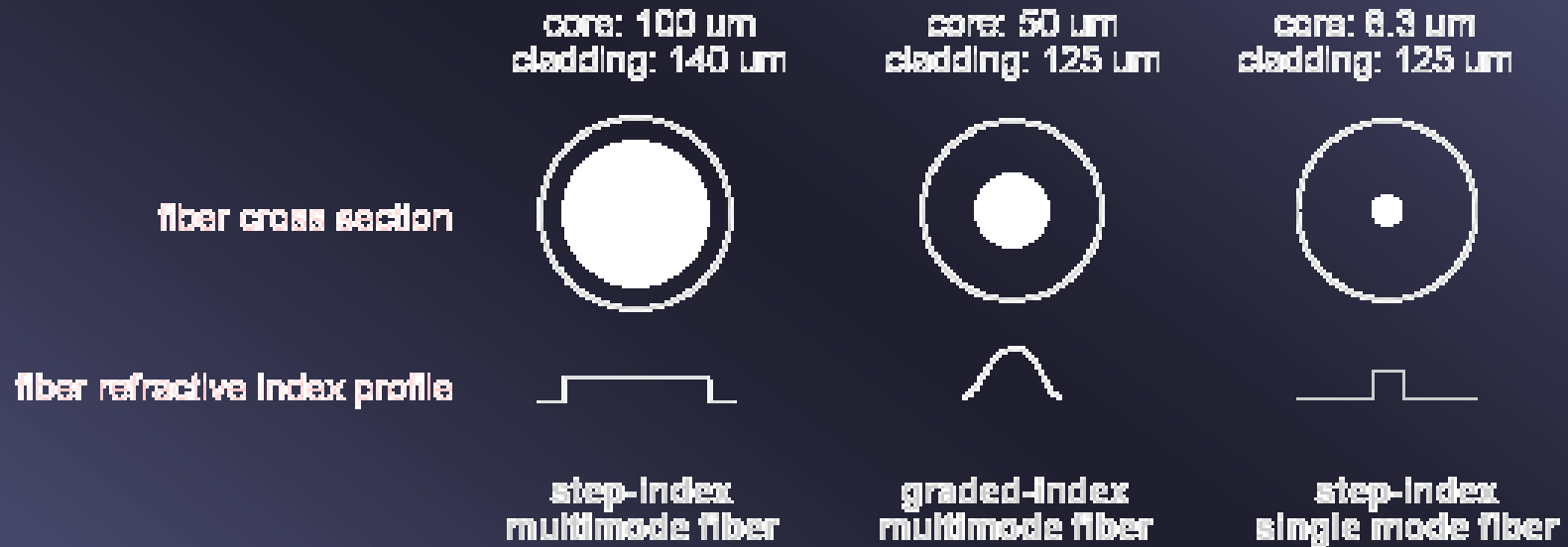


$$n_1 > n_2$$

The fiber



Index profiles

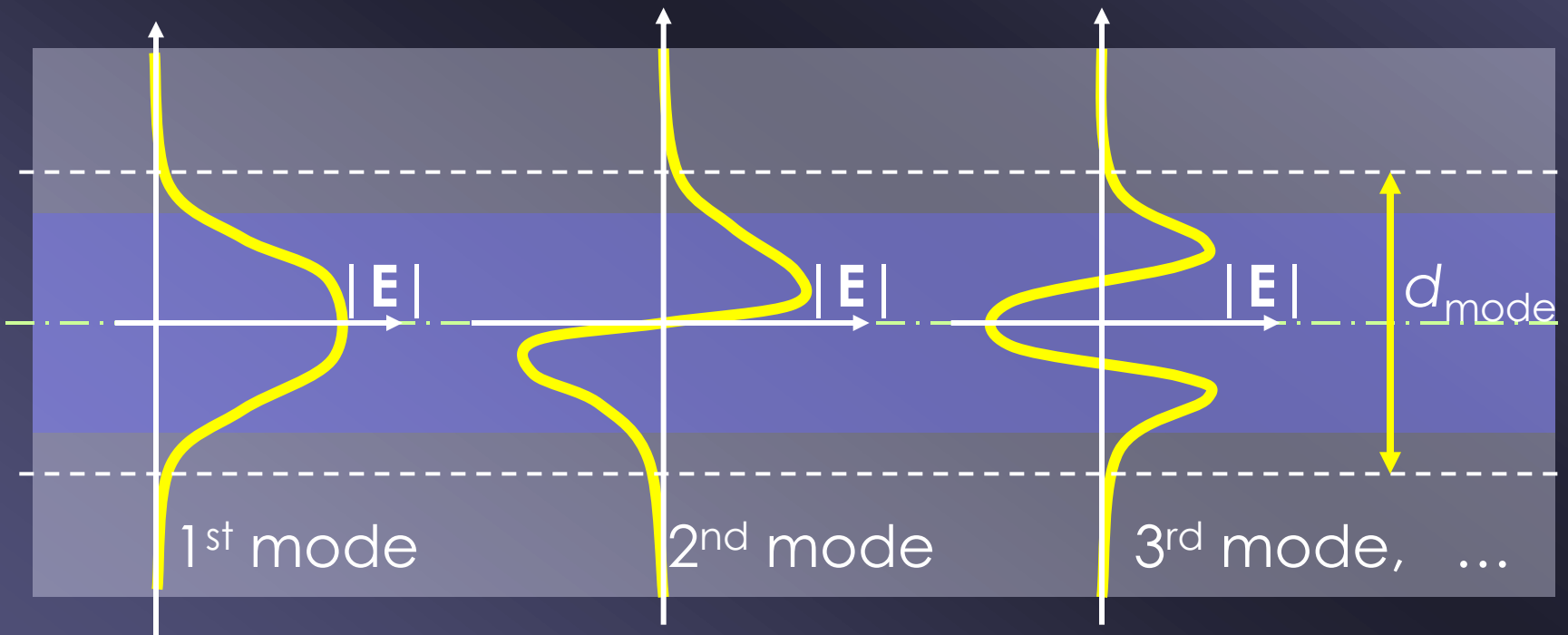


Waveguide (cylindrical) with possible propagating modes

- solution of the electromagnetic wave equation with cylindrical boundary conditions
 - Bessel (and Hankel) functions along the radius
 - propagating waves along the axis

Mode field diameter

- light slightly „penetrates” to the cladding



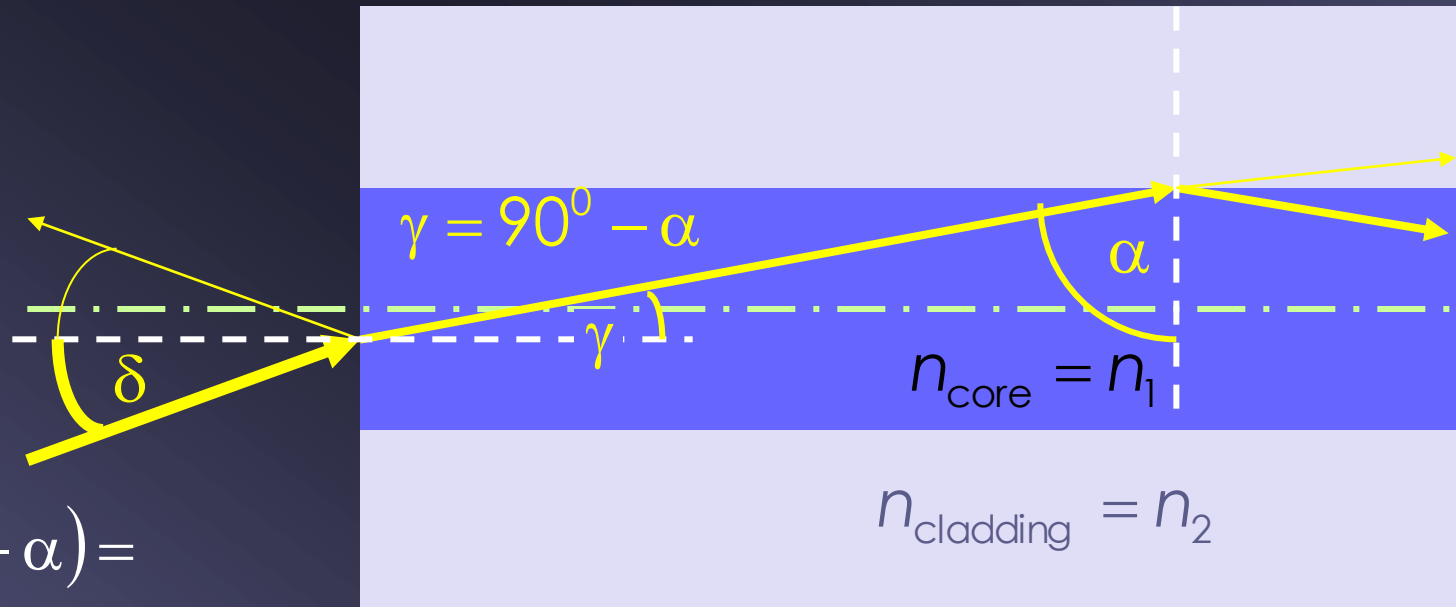
Geometrical optics point of view

- ray bouncing in the cylinder, reflecting (or bending) at the boundaries

$$\frac{\sin \delta}{\sin \gamma} = \frac{n_{\text{core}}}{n_{\text{air}}}$$

$$\sin \delta = n_1 \cdot \sin \gamma$$

$$\sin \gamma = \sin(90^\circ - \alpha) = \cos \alpha$$



Geometrical optics point of view

- ray bouncing in the cylinder, reflecting (or bending) at the boundaries

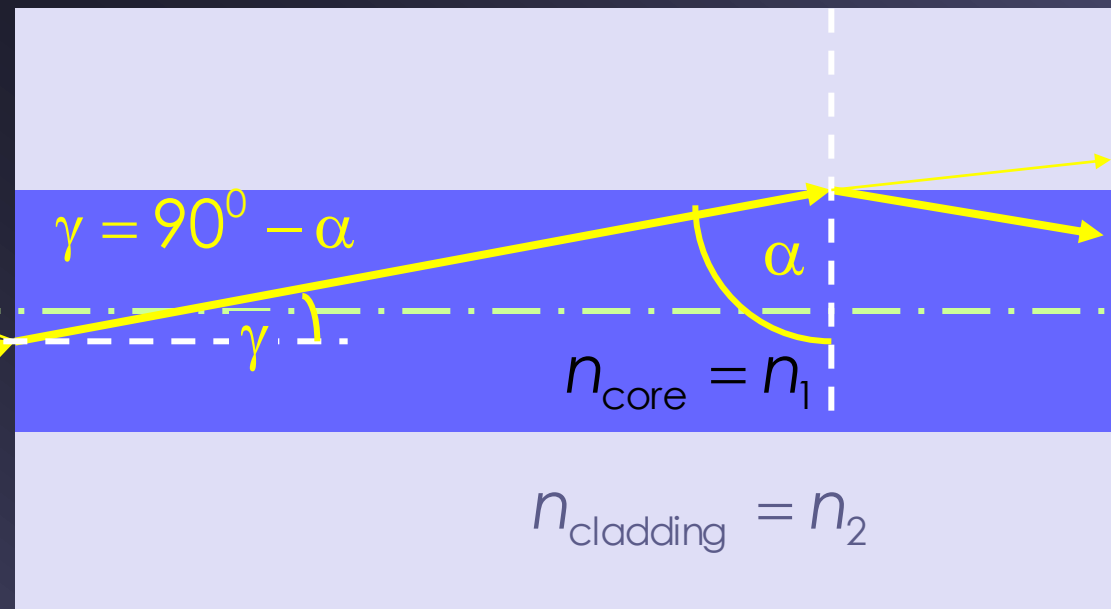
$$\frac{\sin \delta}{\sin \gamma} = \frac{n_{\text{core}}}{n_{\text{air}}}$$

$$\sin \delta = n_1 \cdot \sin \gamma$$

$$\sin \delta = n_1 \cdot \cos \alpha$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$

$$\sin \delta = n_1 \sqrt{1 - \sin^2 \alpha}$$



$$n_{\text{cladding}} = n_2$$

$$n_{\text{air}} \approx 1$$

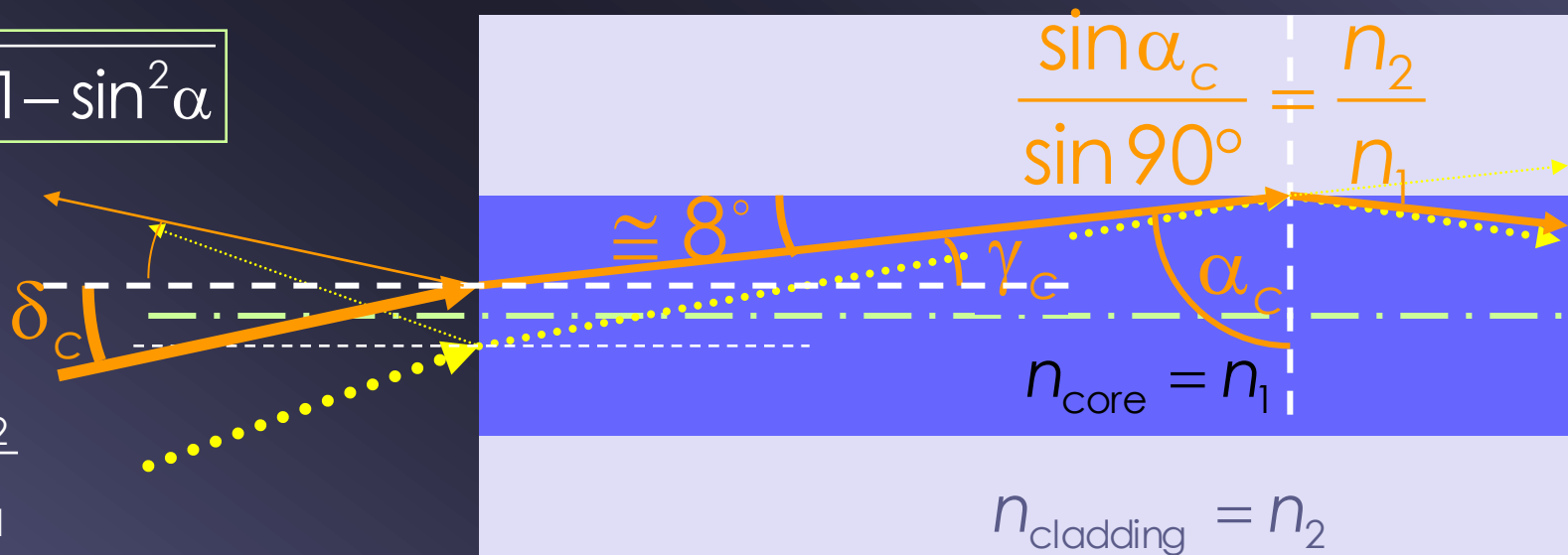
Geometrical optics point of view

- ray bouncing in the cylinder, reflecting (or bending) at the boundaries

$$\sin \delta = n_1 \sqrt{1 - \sin^2 \alpha}$$

$$\sin \alpha_c = \frac{n_2}{n_1}$$

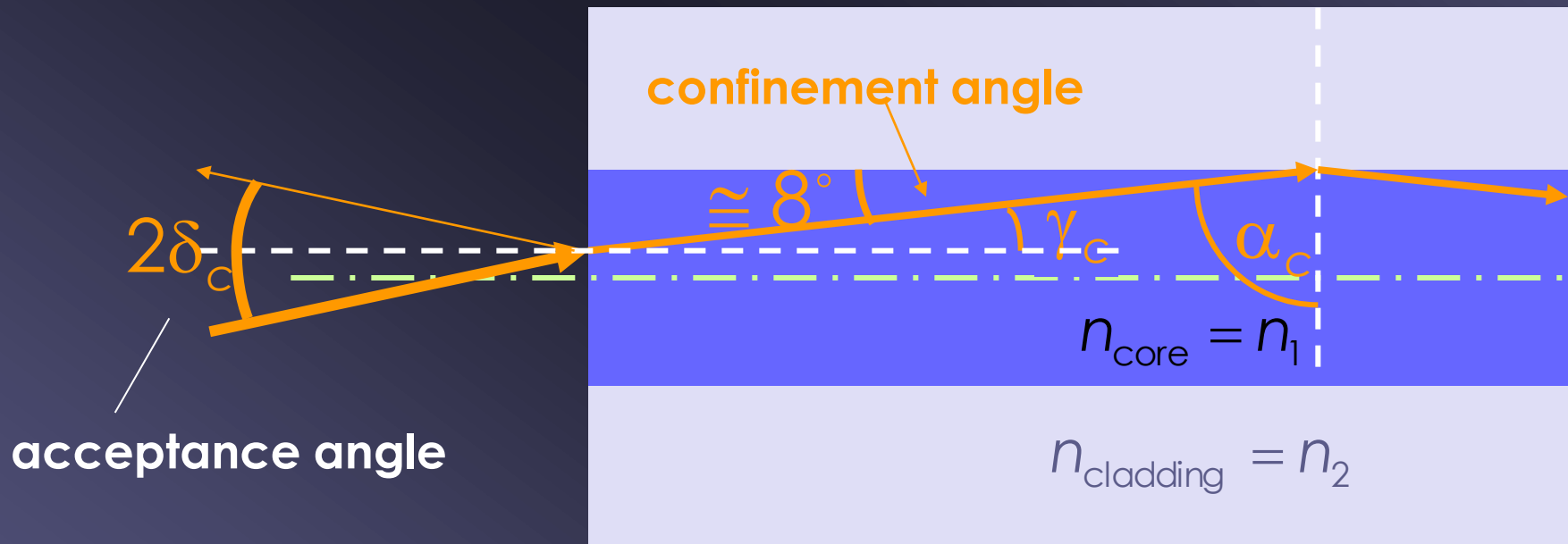
$$\sin \delta_c = NA = \sqrt{n_1^2 - n_2^2}$$



Geometrical optics point of view

- numerical aperture

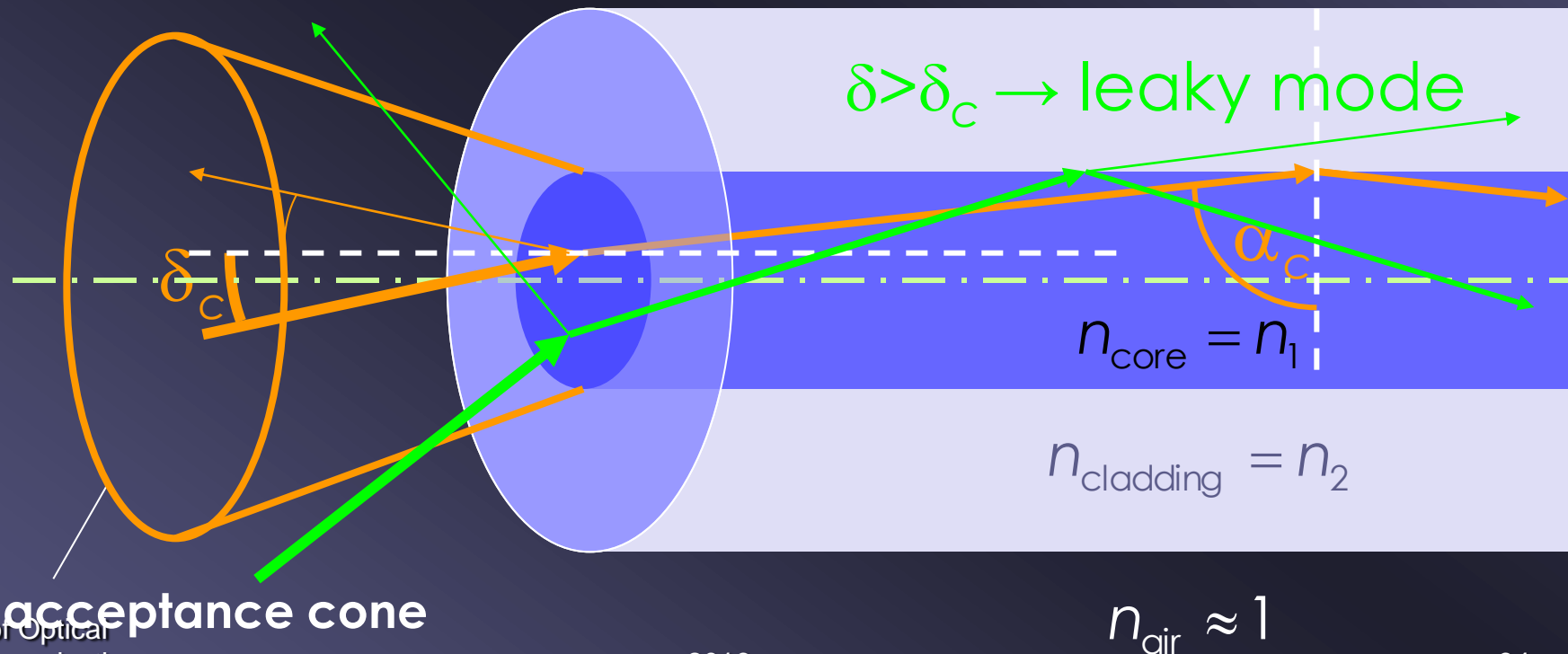
$$\sin \delta_c = NA = \sqrt{n_1^2 - n_2^2}$$



Geometrical optics point of view

- numerical aperture

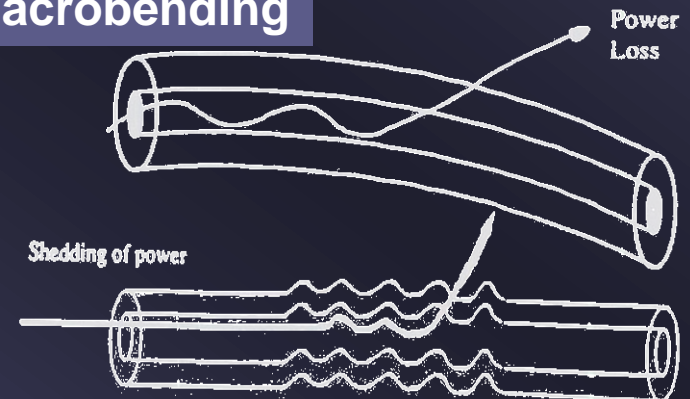
$$\sin \delta_c = NA = \sqrt{n_1^2 - n_2^2}$$



Fiber attenuation

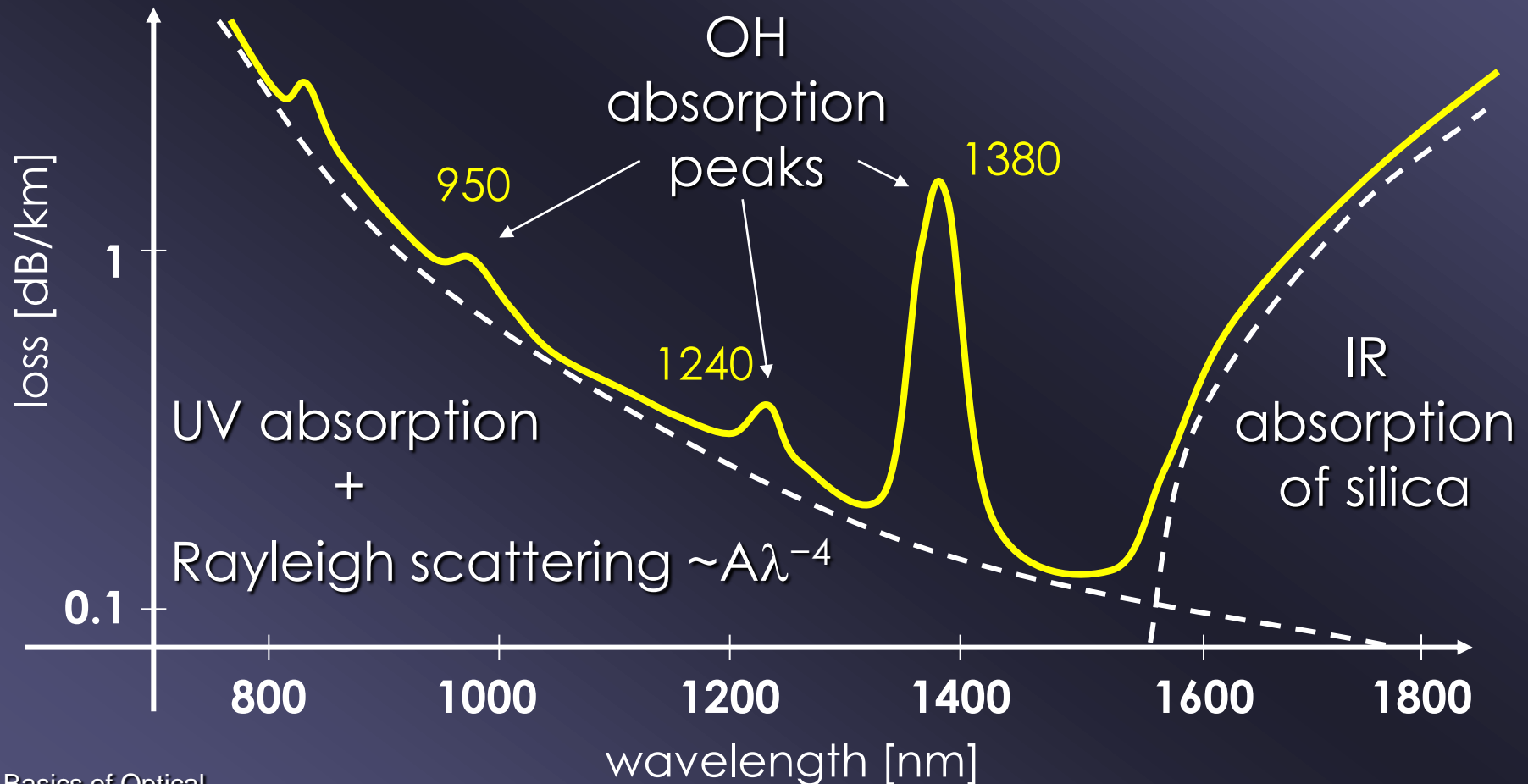
- absorption – cumulative
 - $(1 - \alpha)^L$
 - depends on the composition
- scattering – cumulative
 - Rayleigh scattering
 - $(1 - S)^L$
 - $A\lambda^{-4}$
- coupling losses – at the ends
- bendings

macrobending

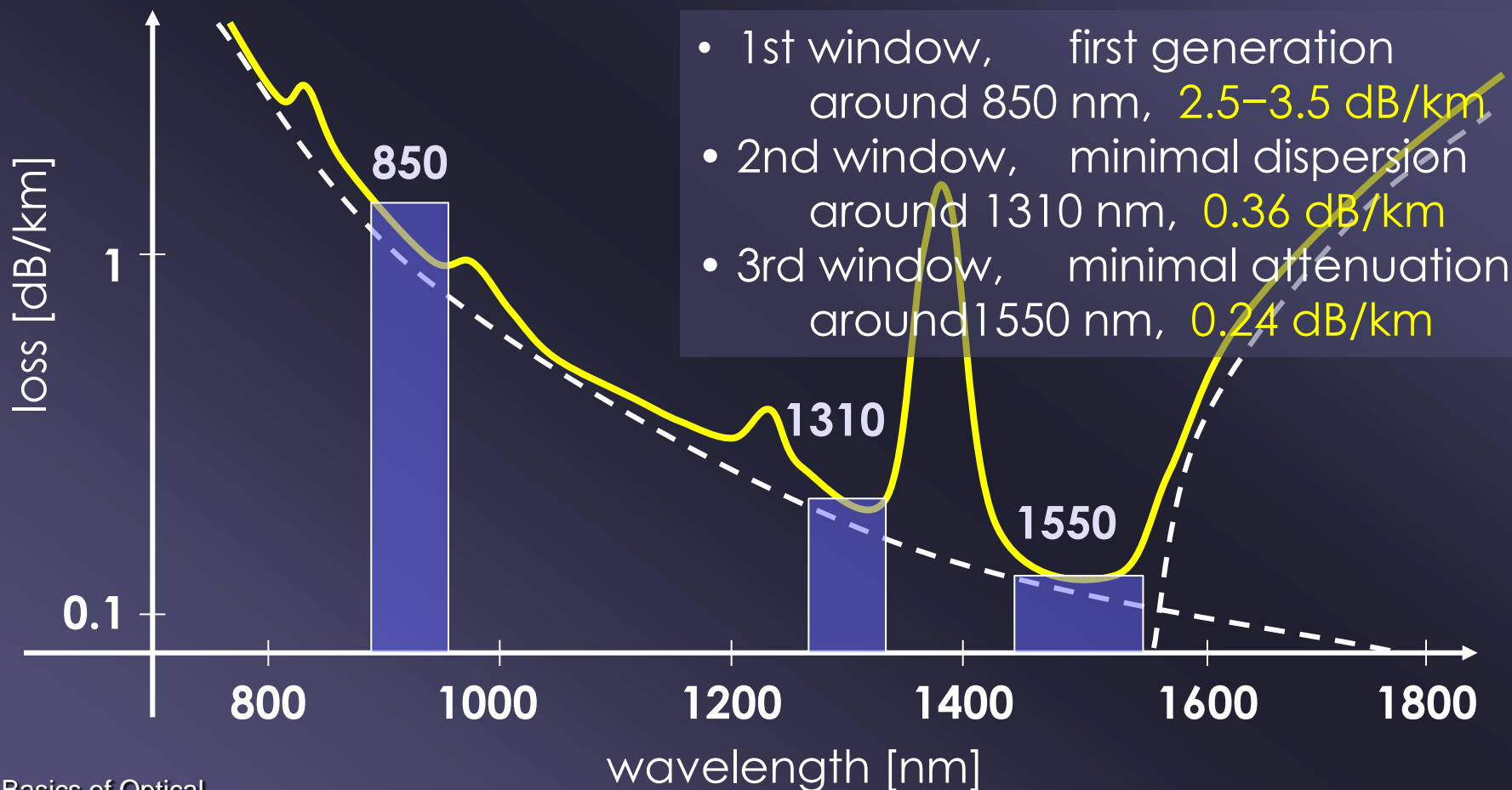


microbending

Total attenuation of silica – attenuation peaks



Total attenuation of silica – optical windows



Fiber dispersion

The spreading out of the light pulses as they propagate in the fiber. (ps/km)

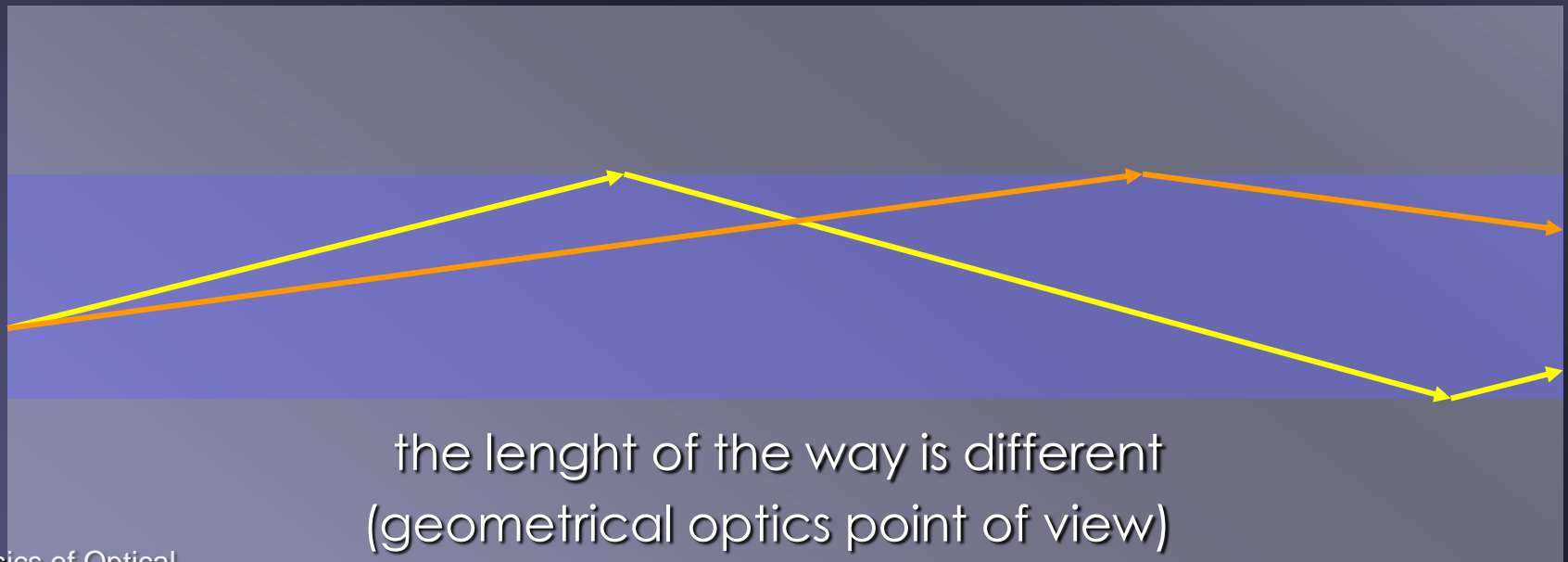
- modal dispersion
- chromatic dispersion
 - material dispersion
 - waveguide dispersion
- polarization mode dispersion

$$\left. \begin{array}{l} \text{material dispersion} \\ \text{waveguide dispersion} \end{array} \right\} \Delta t_{\text{chr}} = \Delta t_{\text{mat}} + \Delta t_{\text{wg}}$$

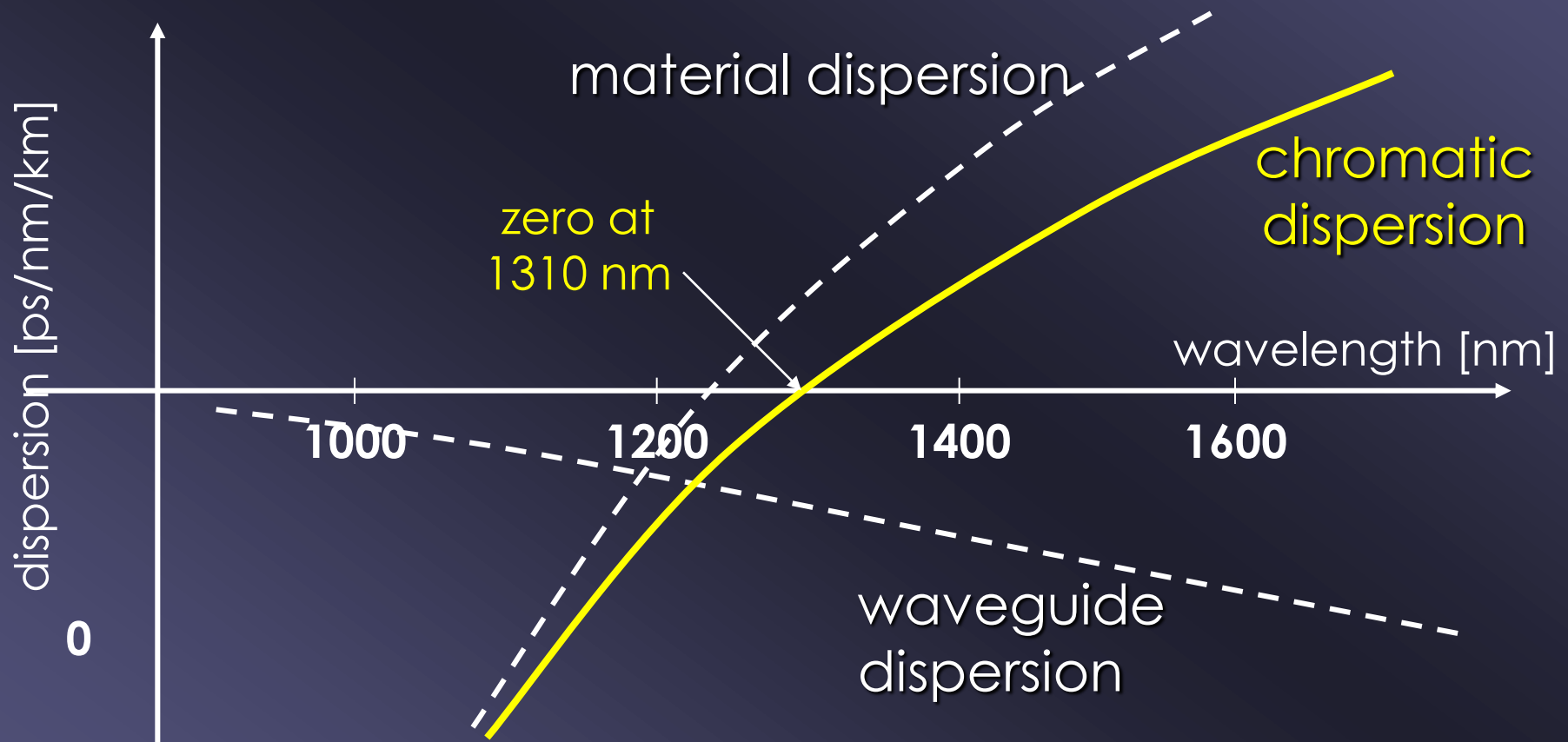
$$\Delta t_{\text{total}} = \sqrt{\Delta t_{\text{mod}}^2 + \Delta t_{\text{chr}}^2 + \Delta t_{\text{pol}}^2}$$

Modal dispersion

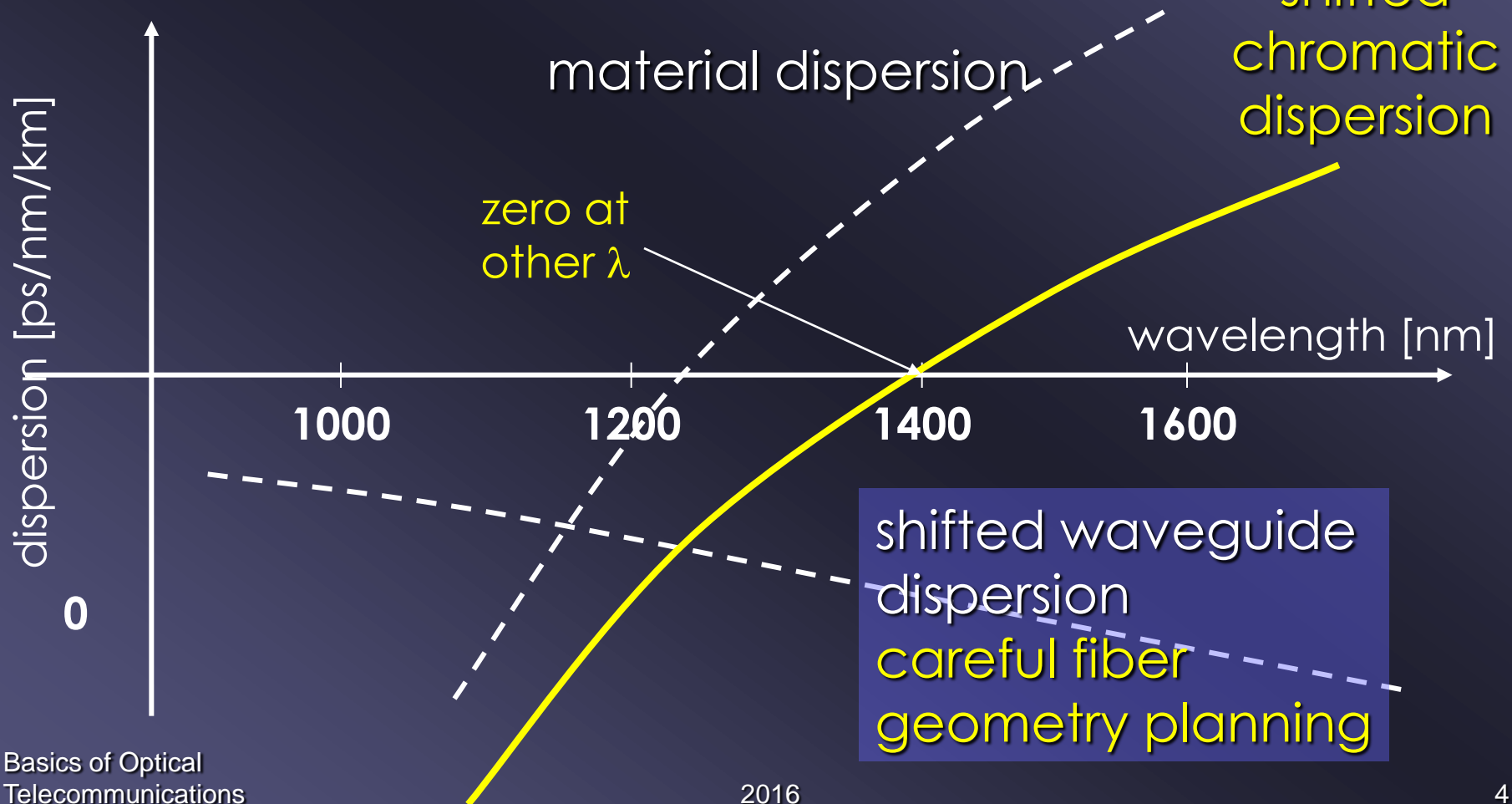
- multimode fibers
- different characteristic velocity for different propagating modes



Chromatic dispersion



Chromatic dispersion shifting



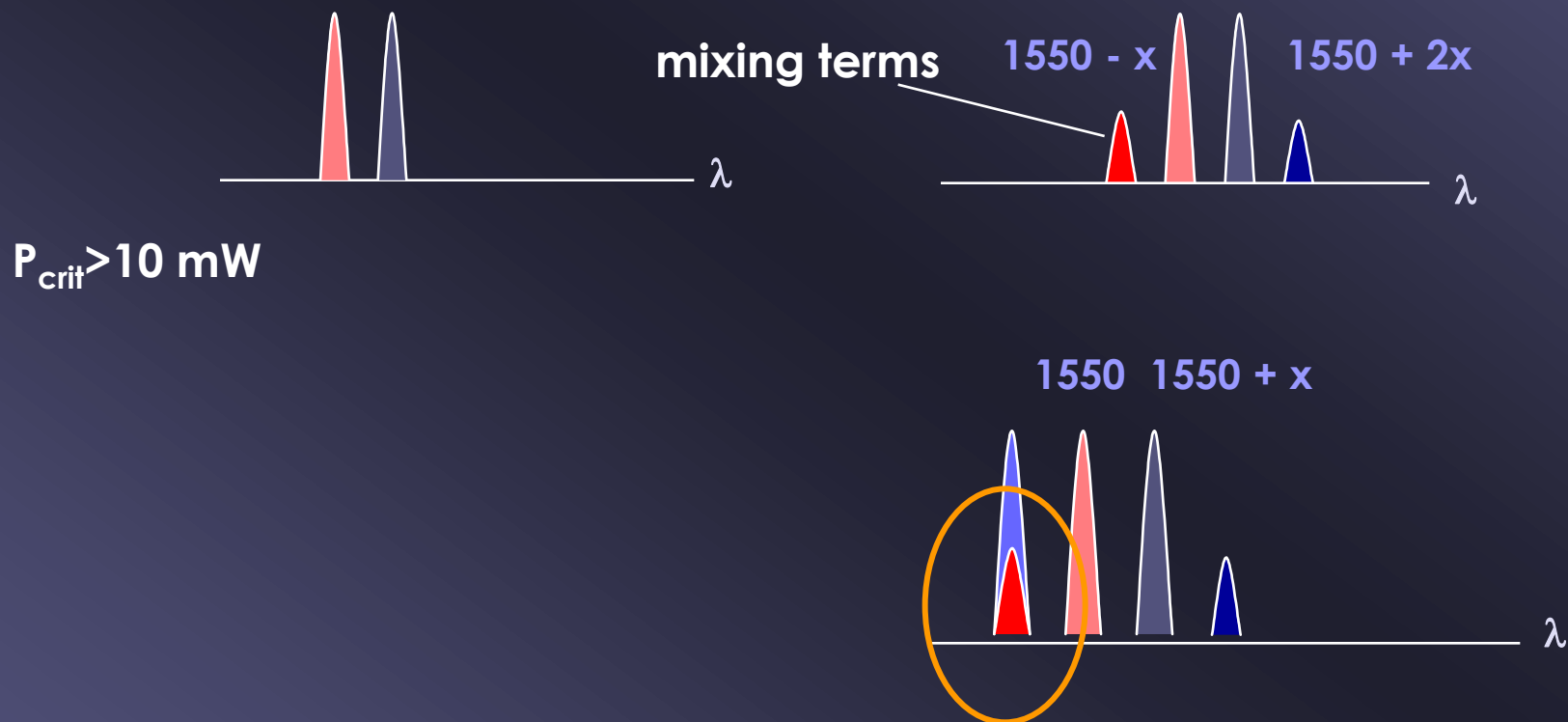
Nonlinear effects

- Brillouin scattering
- self-phase modulation
- cross-phase modulation
- four-wave mixing
- Raman scattering

- Brillouin scattering:
 - acoustic vibrations caused by electro-magnetic field
(e.g. the light itself, if $P > 3\text{mW}$)
 - acoustic waves generate refractive index fluctuations
 - scattering on the refraction index waves
 - the frequency of the light is shifted slightly direction dependently (~ 11 GHz backw.)
 - longer pulses – stronger effect

- Raman scattering:
 - optical phonons (vibrations) caused by electromagnetic field and the light can exchange energy (similar to Brillouin but not acoustical phonons)
- Stimulated Raman and Brillouin scattering can be used for amplification
- Self-phase and cross-phase modulation
- Four-wave mixing

Four-wave mixing



- (Pockels effect:
 - refractive index change due to external electronic field
 - $\Delta n \sim |\mathbf{E}|$ - a linear effect)

• Kerr effect:

- the refractive index changes in response to an electromagnetic field
- $\Delta n = K \lambda |\mathbf{E}|^2$
- light modulators up to 10 GHz
- can cause self-phase modulation, self-induced phase and frequency shift, self-focusing, mode locking
- can produce solitons with the dispersion

- Kerr effect:
 - the polarization vector

$$P_i = \varepsilon_0 \sum_{j=1}^3 \chi_{ij}^{(1)} E_j + \varepsilon_0 \sum_{j=1}^3 \sum_{k=1}^3 \chi_{ijk}^{(2)} E_j E_k + \varepsilon_0 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{l=1}^3 \chi_{ijkl}^{(3)} E_j E_k E_l + \dots$$

Pockels

Kerr

- if $\mathbf{E} = \mathbf{E}_\omega \cos(\omega t)$, the polarization in first order is

$$\mathbf{P} \cong \varepsilon_0 \left(\chi^{(1)} + \chi^{(3)} |\mathbf{E}_\omega|^2 \right) \cdot \mathbf{E}_\omega \cos(\omega t)$$

- Kerr effect:

$$\mathbf{P} \cong \varepsilon_0 \left(\chi^{(1)} + \chi^{(3)} |\mathbf{E}_\omega|^2 \right) \cdot \mathbf{E}_\omega \cos(\omega t)$$

- the susceptibility

$$\chi = \chi^{(1)} + \frac{3}{4} \chi^{(3)} |\mathbf{E}_\omega|^2$$

- the refractive index

$$n = n_0 + \frac{3}{8n_0} \chi^{(3)} |\mathbf{E}_\omega|^2 = n_0 + n_2 I$$

- n_2 is mostly small, large intensity is needed (silica:
 $n_2 \approx 10^{-20} \text{m}^2/\text{W}$, $I \approx 10^9 \text{W}/\text{cm}^2$)

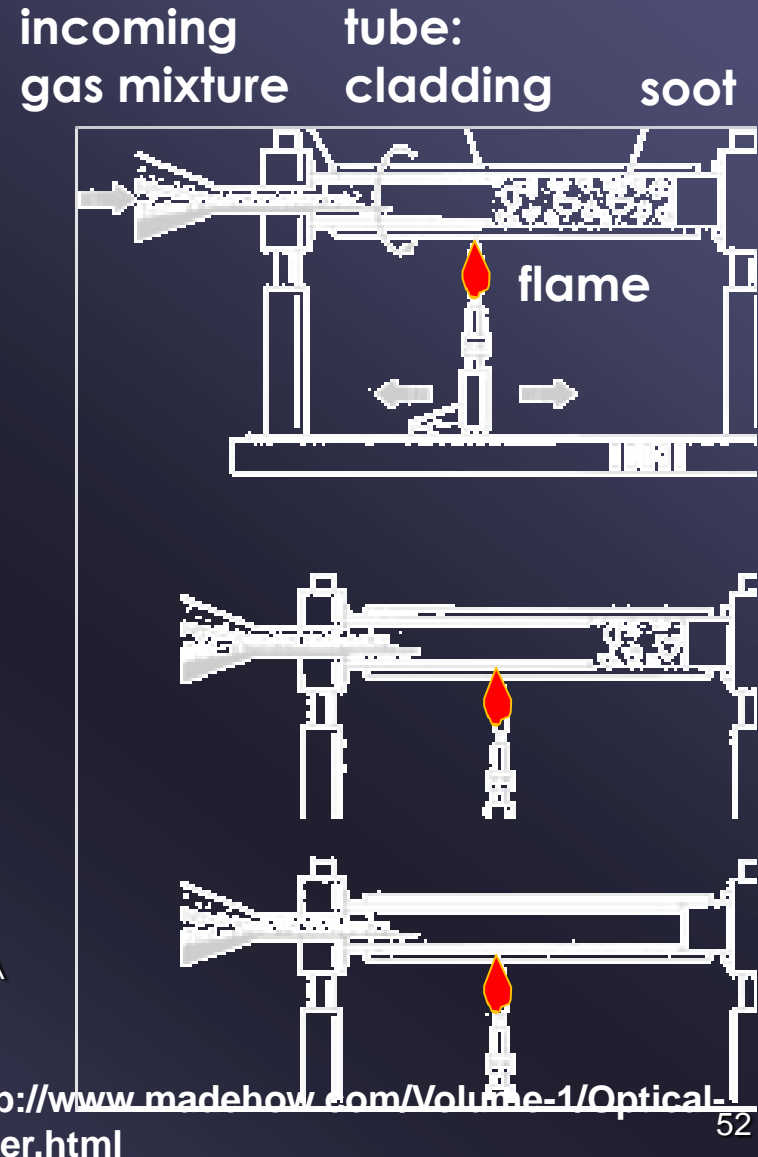
• Gordon-Haus jitter:

- a timing jitter originating from fluctuations of the center frequency of the (soliton) pulse
- noise in fiber optic links caused by periodically spaced amplifiers
- the amplifiers introduce quantum noise, this shifts the center frequency of the pulse
- the behavior of the center frequency modeled as random walk

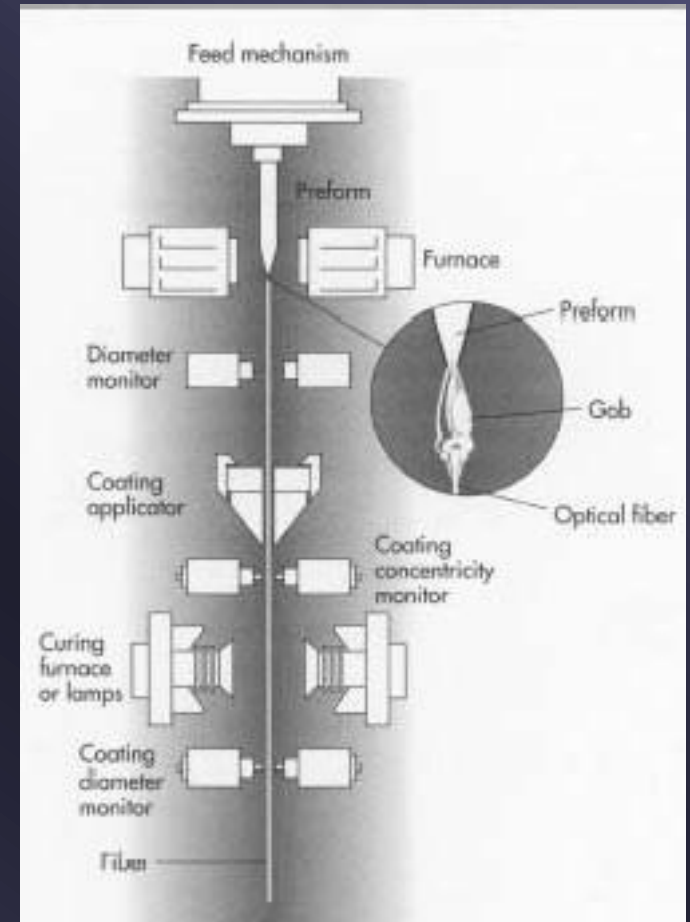
- Gordon-Haus jitter:
 - dominant in long-haul data transmission
 - $\sim L^3$,
 - can be suppressed by
 - regularly applied optical filters
 - amplifiers with limited gain bandwidth
 - can also take place in mode-locked lasers

- Vapor deposition, usually starting from the cladding
 - cleaning
 - burning SiCl_4 + dopants in oxygen inside the cladding
 - preform collapsing – slow heating of the cladding and the glassy soot → they melt together and collapse into solid rod

see: <https://www.youtube.com/watch?v=uSnjo5tOGQA>

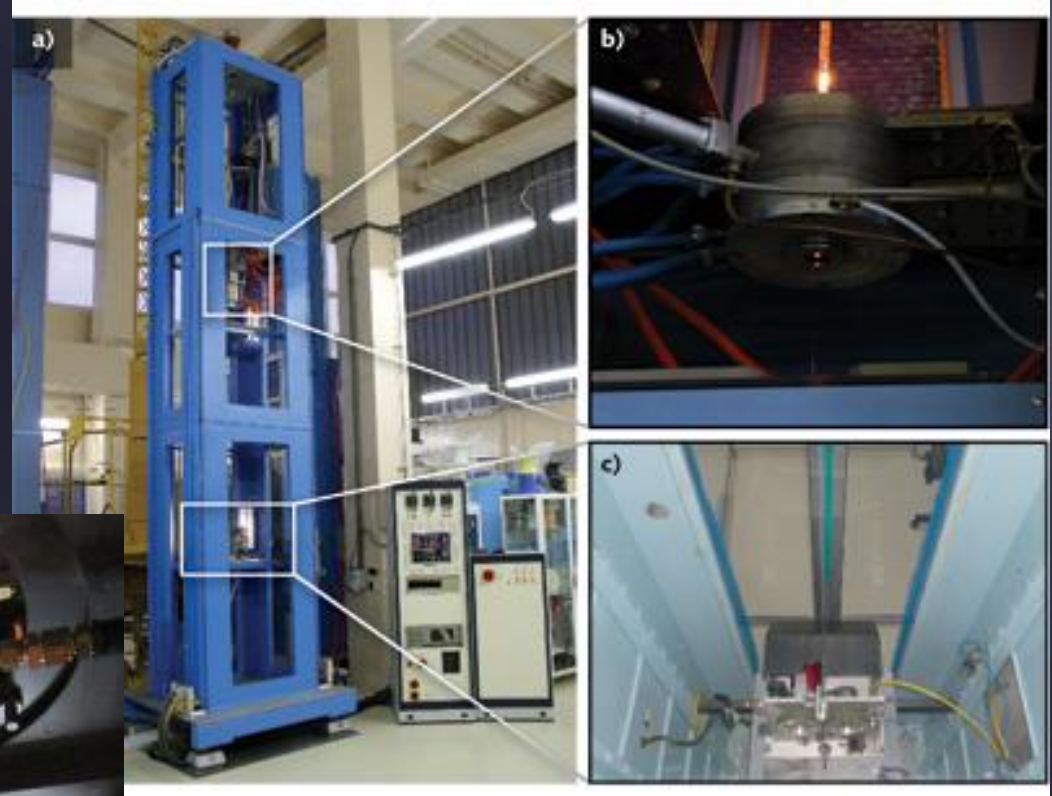


- Fiber pulling from preform
 - quartz furnace at the top of the pulling tower
 - forming of a droplet
 - droplet pulls the fiber
 - after achieving the sufficient diameter/length the gob is cut
 - reel pulls the fiber



Fabrication of fibers

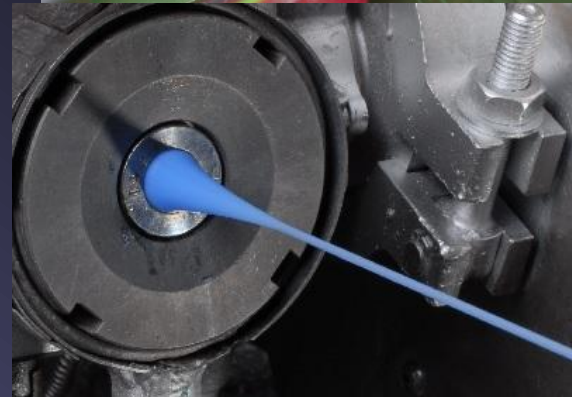
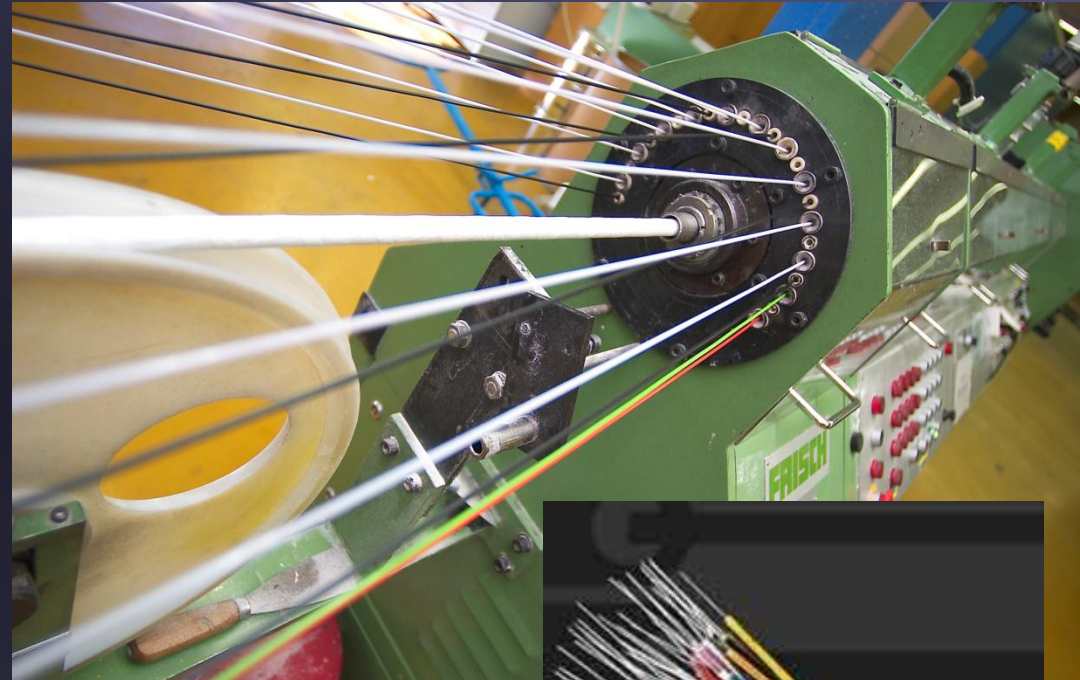
<http://www.orc.soton.ac.uk/silicafibrefacilities.html>



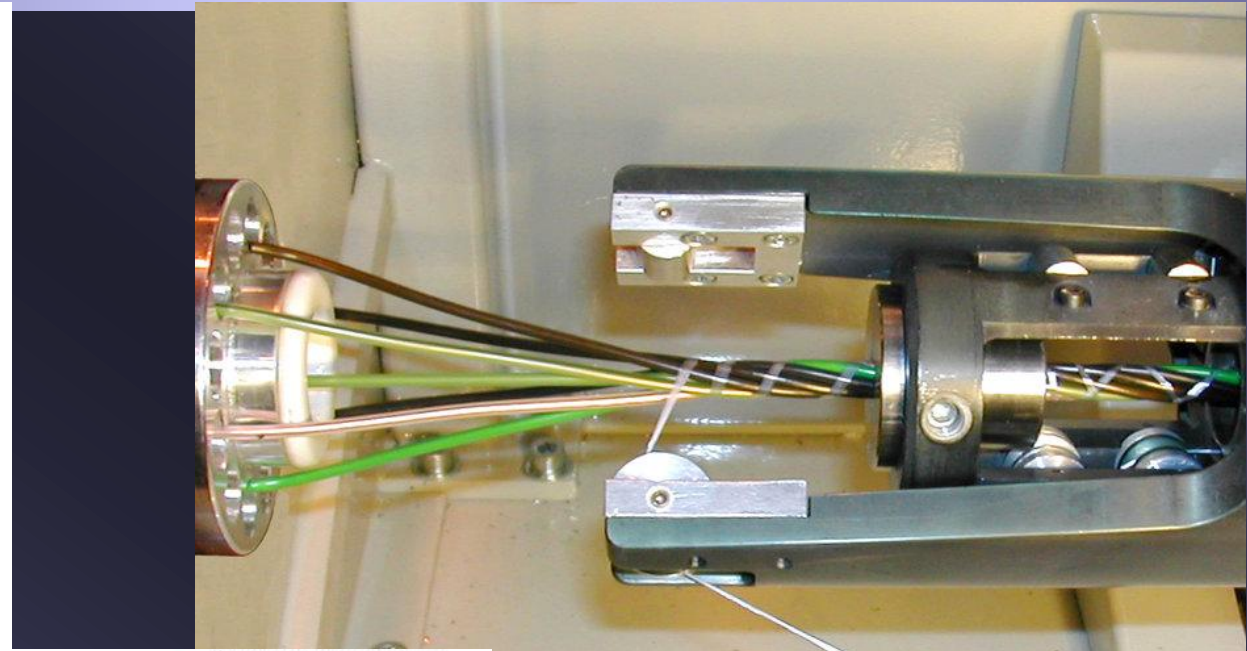
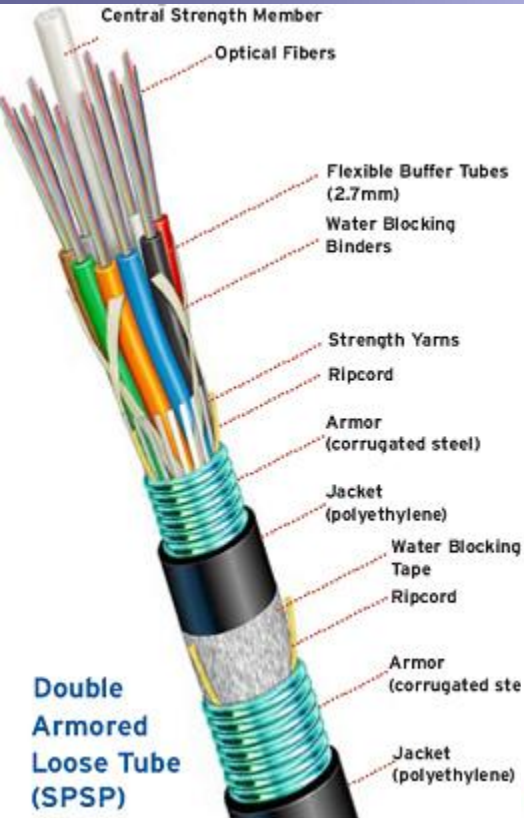
<http://www.laserfocusworld.com/articles/print/volume-49/issue-12/features/optical-fiber-manufacturing-stack-and-draw-technique-creates-ultras-small-diameter-endoscopes.html>

Cable production

- From reels of fiber with primary coating
- SZ twist
- sheath extrusion
- various fillings
- various protective layers: armors, jackets, sheaths
- various strength members
- ribbon or cylindrical



Cable production



- Indoor
 - Patch, Switch, Pigtail
 - FTTH loose cables
- Outdoor
 - loose or tight
 - ribbon or cylindrical
 - air
 - self sustaining or not
 - underground, or underwater
 - armored or not

- PC or APC
- ferrule
- cylindrical or rectangular housing

- Operation of lasers
 - Properties
 - applications
- Atomic energy levels
- Population inversion
- Energy bands in solid states
- Heterojunctions in semiconductors
- Quantum well lasers
- Vertical cavity surface emitting lasers

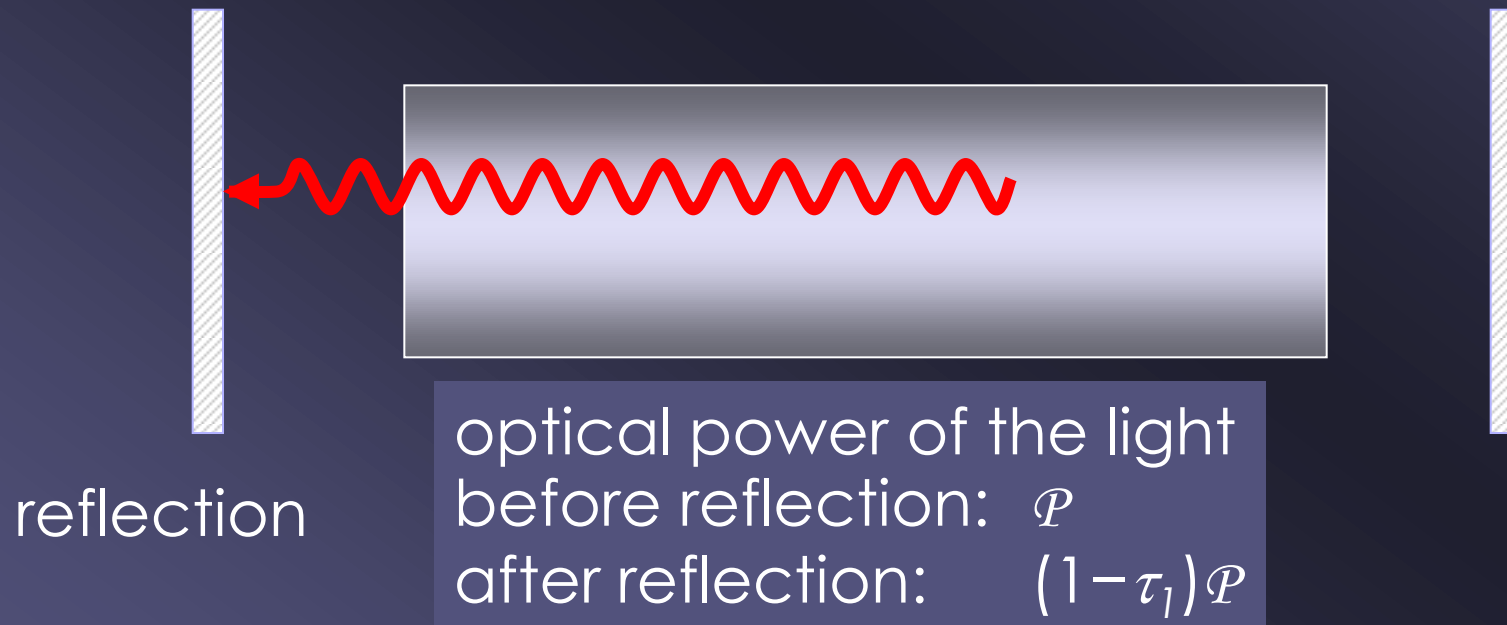
- Monochromatic light – small bandwidth
- Small divergence – narrow and directed beam
- Coherent beam – all photons have nearly the same phase

- Usually not too high power, but
- High power density
- Not an effective energy transformer

- Materials processing – cutting, drilling, welding, heat treating, ...
- Reading optical signs – CD, barcode, ...
- Graphics – printers, color separators, printing plate makers, ...
- Laboratory, measurements
- Medicine – bloodless scalpel, tumor destroying, ...
- Military – target designators, finders, ...
- Telecommunications

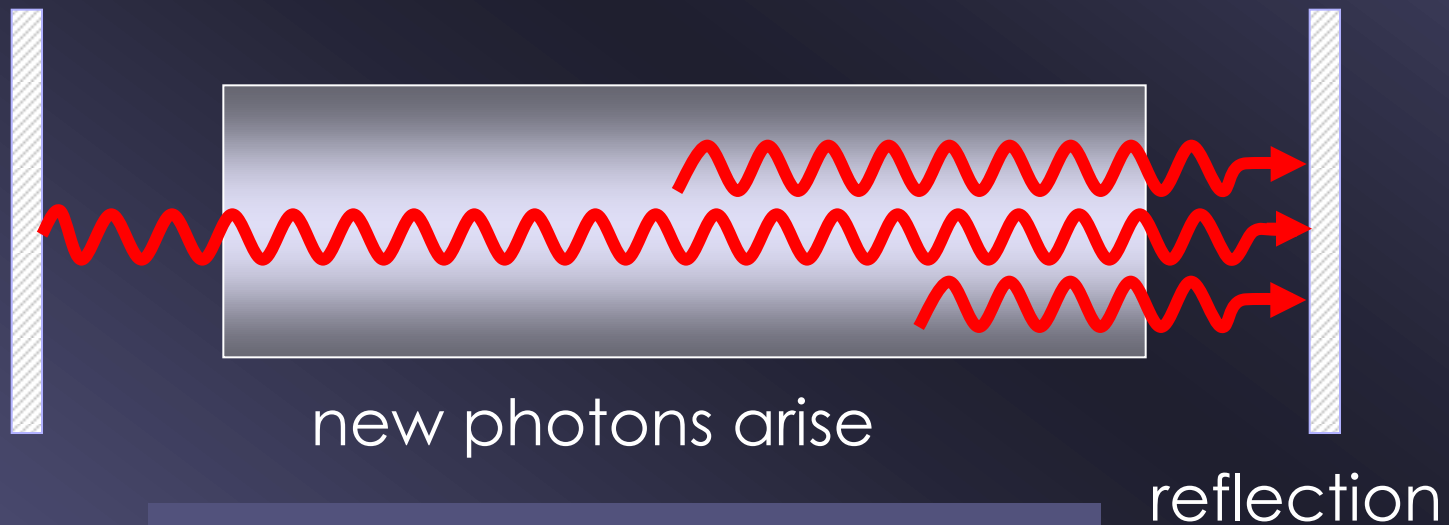
What is needed for laser operation

- Laser gain – an optical amplifier
- Optical resonator – positive feedback



What is needed for laser operation

- Laser gain – an optical amplifier
- Optical resonator – positive feedback

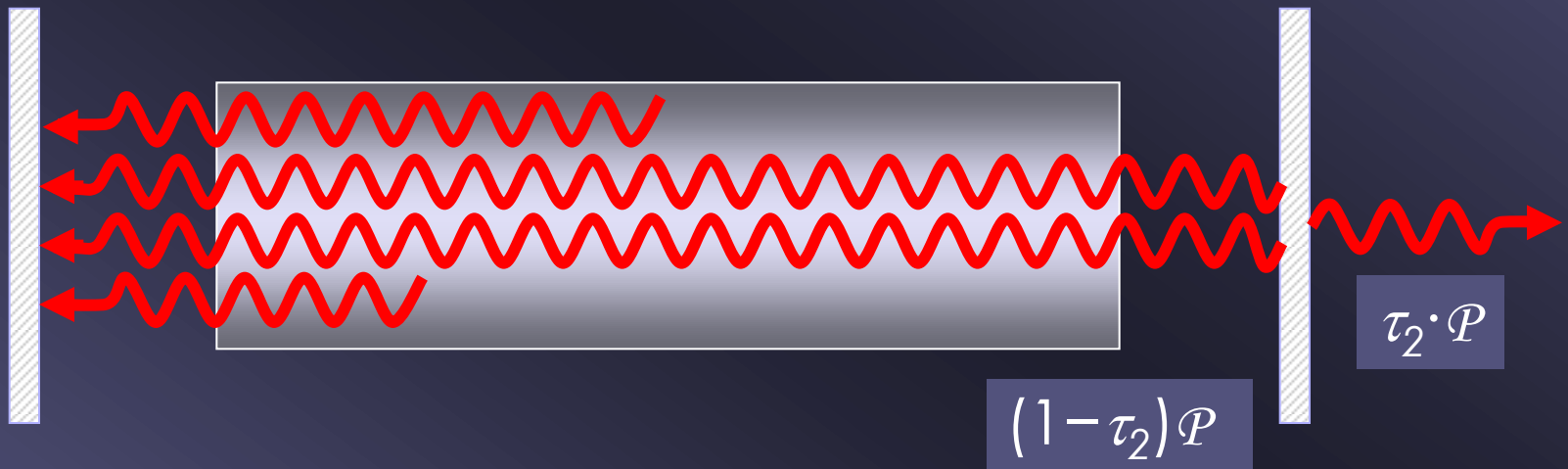


optical gain in the amplifier:

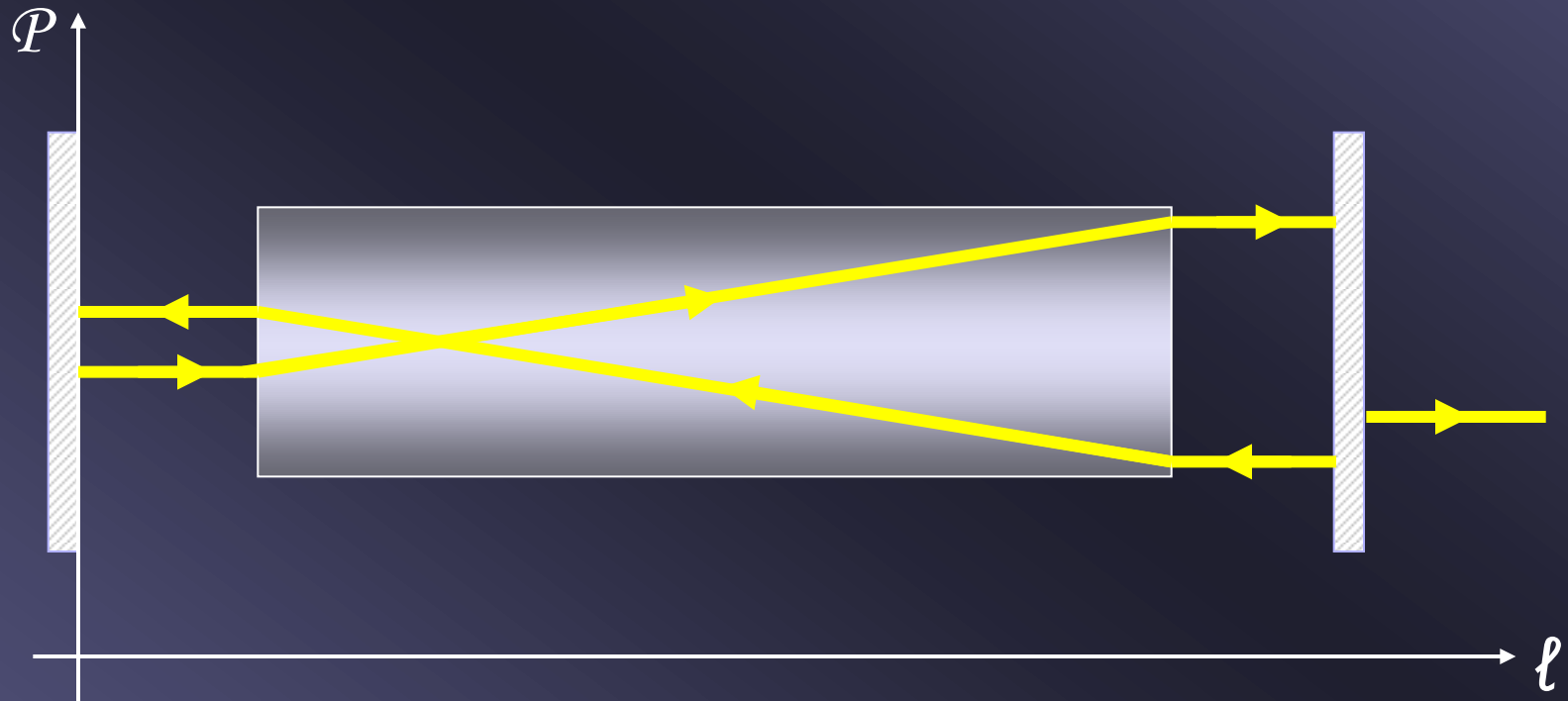
$$\mathcal{P} \longrightarrow g \cdot l \cdot \mathcal{P}$$

What is needed for laser operation

- Laser gain – an optical amplifier
- Optical resonator – positive feedback



In equilibrium the gain and the losses have to be the same: the power of the light varies as



The solution of the Schrödinger equation

$$\hat{H}\Psi = E\Psi$$

results in

- quantized eigenenergies
- corresponding wave functions

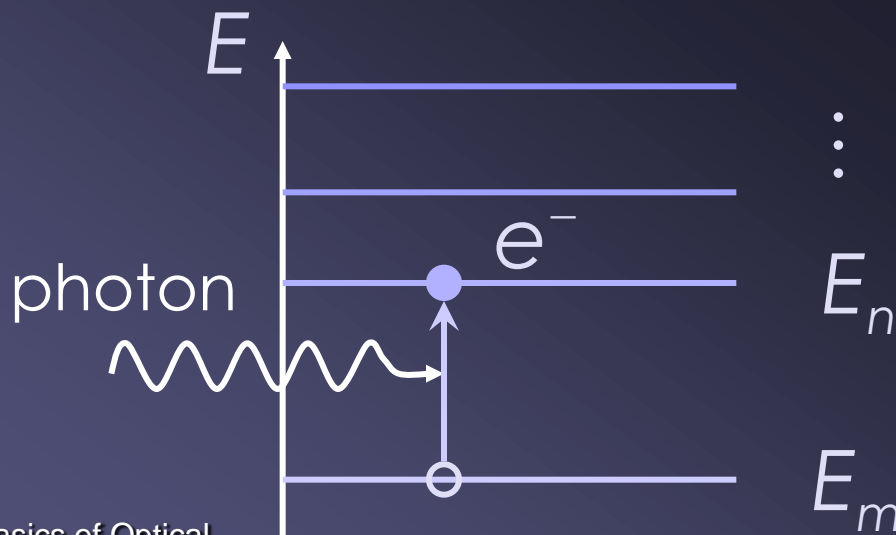


Atomic energy levels

If a photon of energy

$$h\nu = E_n - E_m$$

interacts with an atom, an electron can be excited from energy level E_m to level E_n



photon absorption
– relative rate:

$$r_{mn} = B_{mn} \cdot f_m (1 - f_n) \cdot \rho(h\nu)$$

Atomic energy levels

An excited electron from energy level E_m can relax to a lower from energy level E_n , releasing a photon of energy

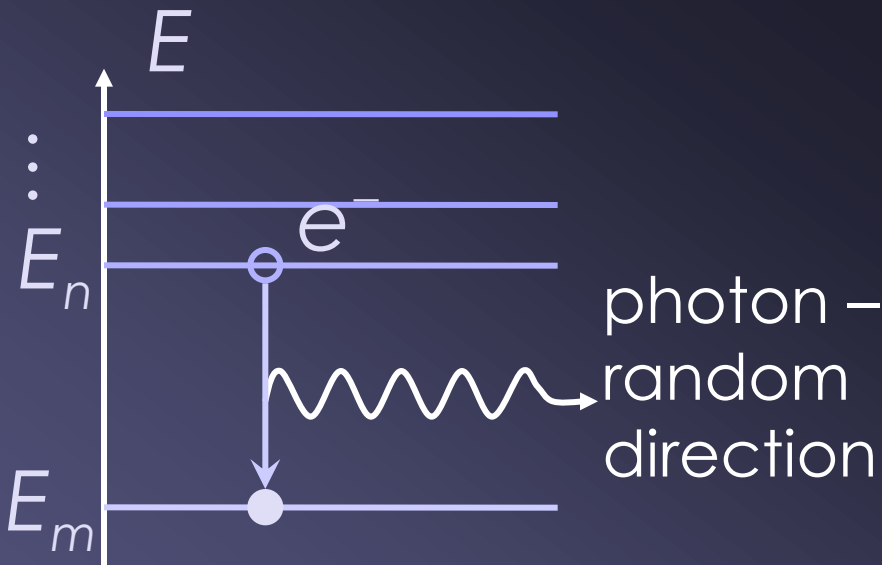
$$h\nu = E_n - E_m$$

spontaneous emission

– relative rate:

$$r_{nm} = A_{nm} \cdot f_n (1 - f_m)$$

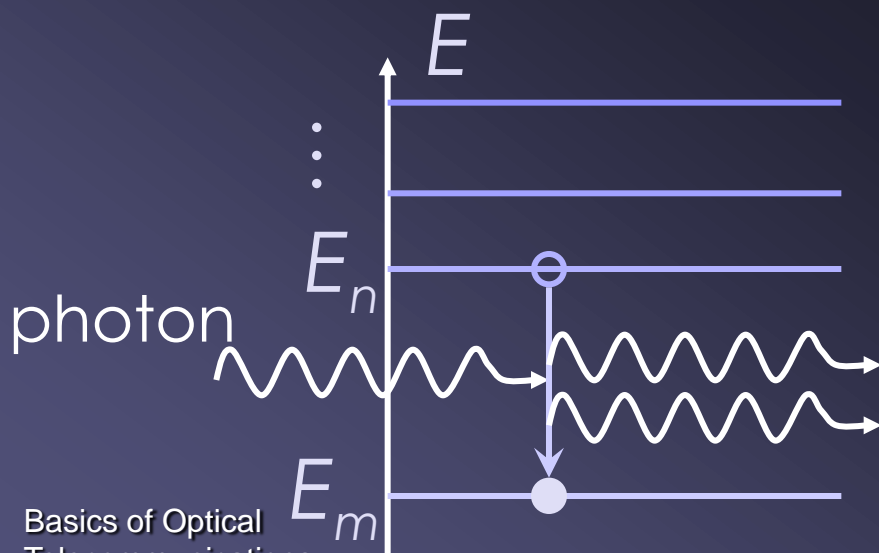
spontaneous lifetime



If a photon corresponding to the energy

$$h\nu = E_n - E_m$$

interacts with an atom which has an excited electron at energy level E_n , it can stimulate the electron to relax to level E_m



stimulated emission

$$r_{mn}^{\text{stim}} = B_{mn} f_m (1 - f_n) \rho(h\nu)$$

2 photons –
same direction,
same phase

Stimulated emission can take place long before the spontaneous lifetime.

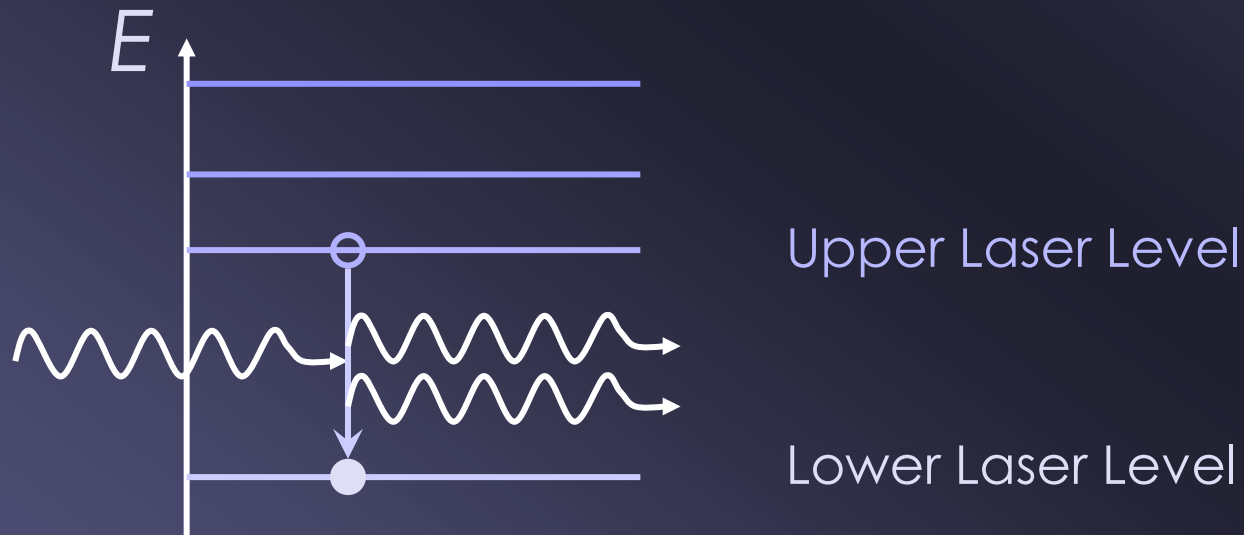
Stimulated emission:

one photon in \implies two photons out

The optical amplifier can be a collection of atoms with lots of electrons excited to the same state (with long spontaneous lifetime).

Light Amplification by Stimulated Emission of Radiation

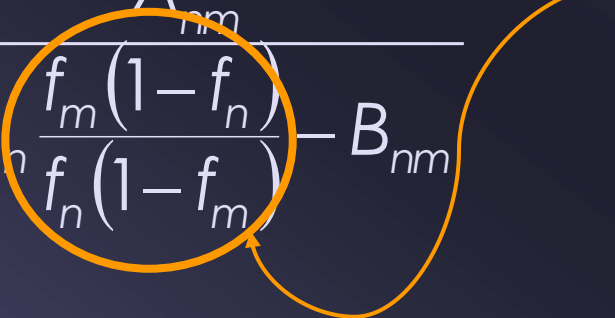
The resonator is usually much longer than the wavelength.



In equilibrium, the relative rates

$$r_{mn} = r_{nm} + r_{nm}^{\text{stim}}$$

Thus the photon density at energy $h\nu$

$$\rho(h\nu) = \frac{A_{nm}}{B_{mn} \frac{f_m(1-f_n)}{f_n(1-f_m)} - B_{nm}}$$


relative
occupation
probability

In thermodynamical equilibrium, the population of the states follow Boltzmann's law

$$N_i = N_0 \cdot e^{-\frac{E_i}{k_B T}}$$

⇒ the relative occupation probability is

$$\exp\left(\frac{E_n - E_m}{k_B T}\right)$$

thus

$$\rho(h\nu) = \frac{A_{nm}}{B_{mn} \cdot \exp\left(\frac{E_n - E_m}{k_B T}\right) - B_{nm}}$$

Comparing the resulting photon density with the black body radiation

$$\rho(h\nu) = \frac{4h\nu^3}{c^2 \cdot \left(\exp \frac{h\nu}{k_B T} - 1 \right)}$$

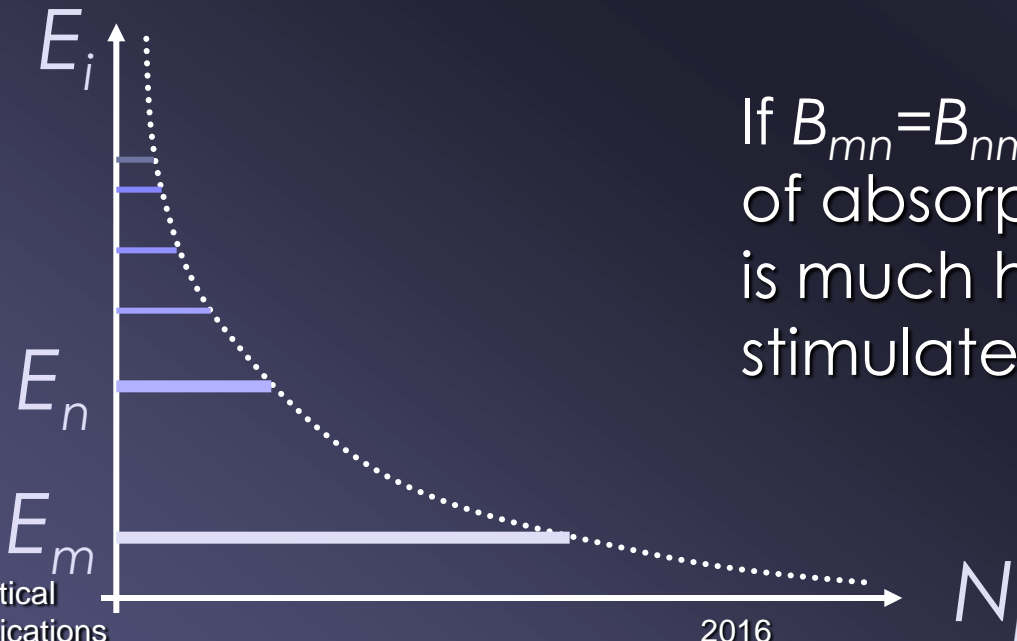
$$\rho(h\nu) = \frac{A_{nm}}{B_{mn} \cdot \exp \frac{E_n - E_m}{k_B T} - B_{nm}}$$

$$B_{mn} = B_{nm}$$

$$\frac{A_{nm}}{B_{nm}} = \frac{4h\nu^3}{c^2}$$

In thermodynamical equilibrium, the population of the states follow Boltzmann's law

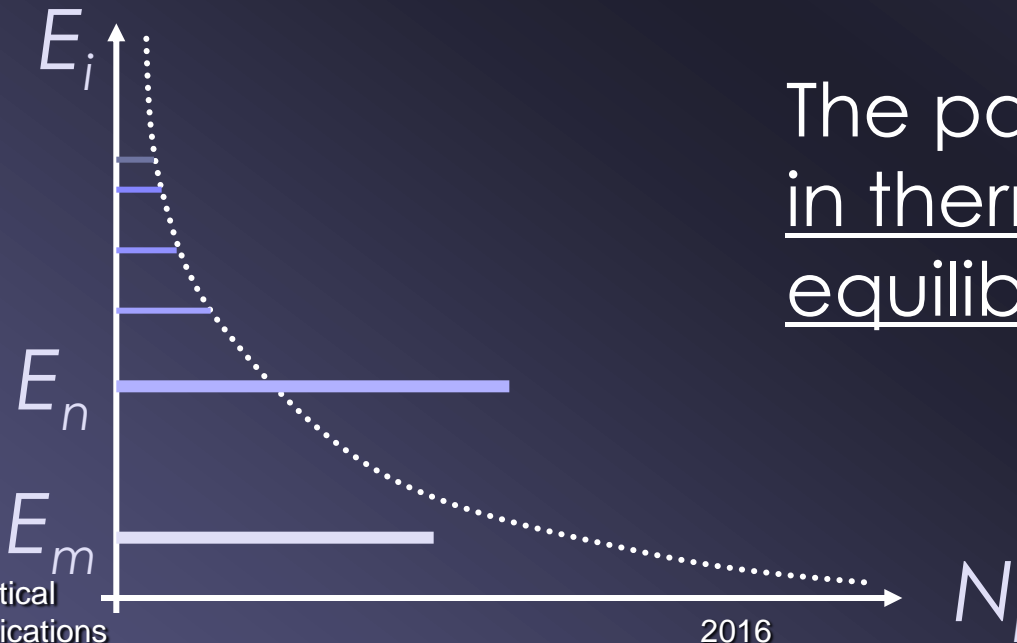
$$N_i = N_0 \cdot e^{-\frac{E_i}{k_B T}}$$



If $B_{mn} = B_{nm}$, the relative rate of absorption in equilibrium is much higher than that of stimulated emission

Population inversion

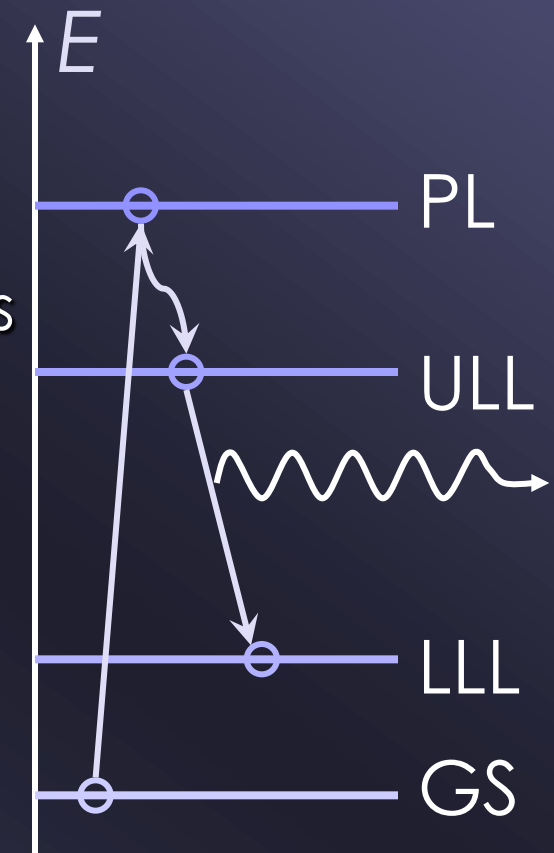
Somehow the number of electrons in the upper laser level is increased \longrightarrow
population inversion occurs.



The particles are not in thermodynamical equilibrium

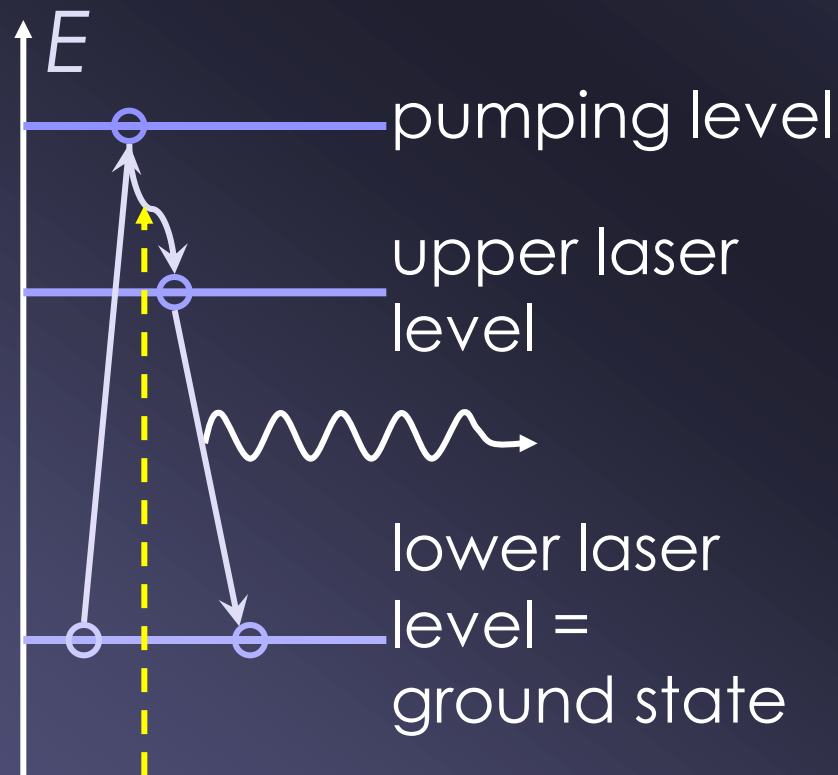
Population inversion is generated by

- exciting the electrons to a level with short spontaneous lifetime above the upper laser level: **pumping**
- from the **pumping level** the electrons relax to the **upper laser level**, which has longer spontaneous lifetime
- electrons accumulate at the upper laser level

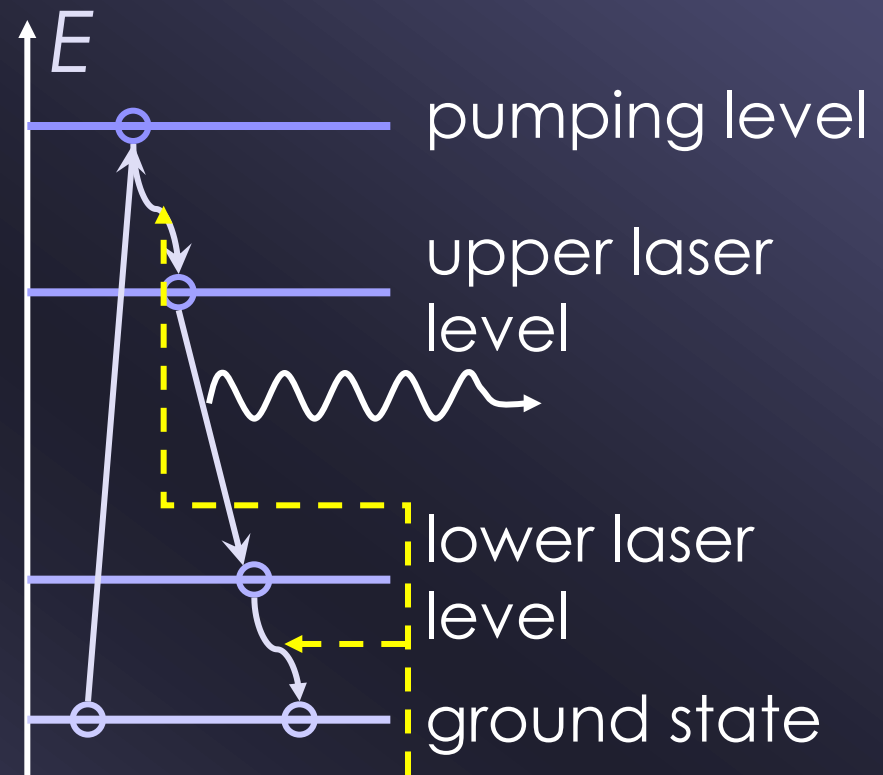


Population inversion

Three-level laser



Four-level laser



short spontaneous lifetime

Inverse population can be generated by

- special filters
- electrical pumping
 - direct electrical discharge
 - radio frequency field
 - electron beam
 - p-n heterostructure
- optical pumping
- chemical pumping
- nuclear pumping

In solids the atomic niveaus broaden \Longrightarrow
energy bands are formed

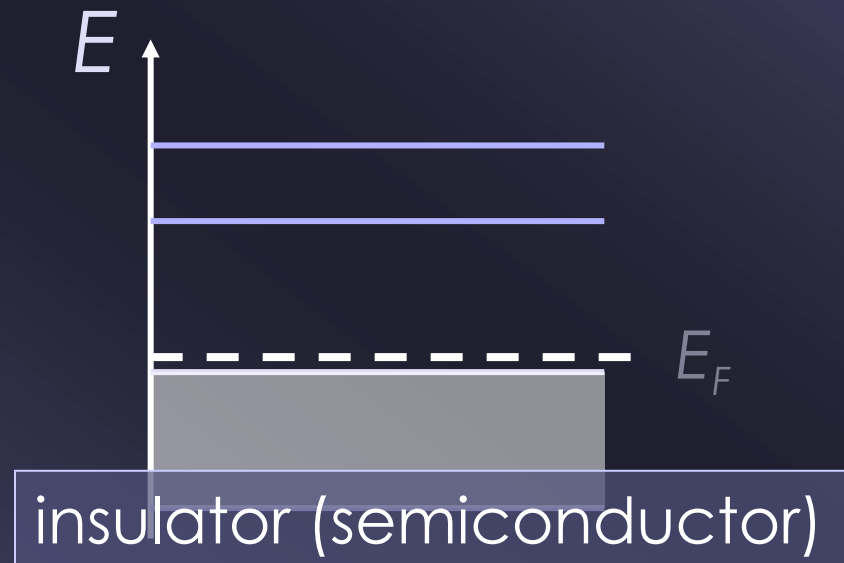
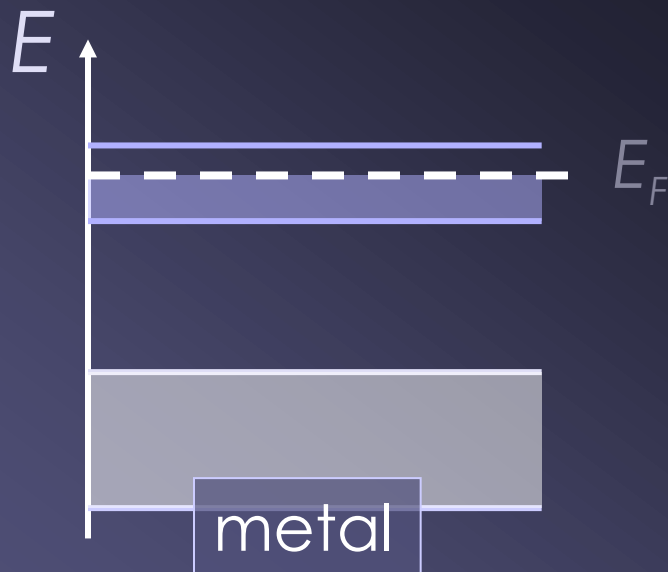
- vibrations (and rotations) in the crystal
- momentum dependence of energy levels
- splitting of degenerate states, ...



⋮
conduction band
bandgap – no electrons are
allowed
valance band
⋮

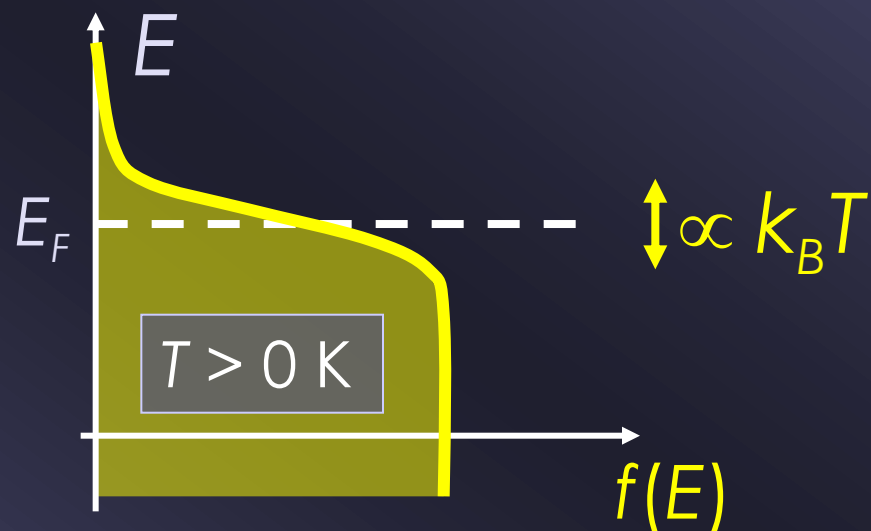
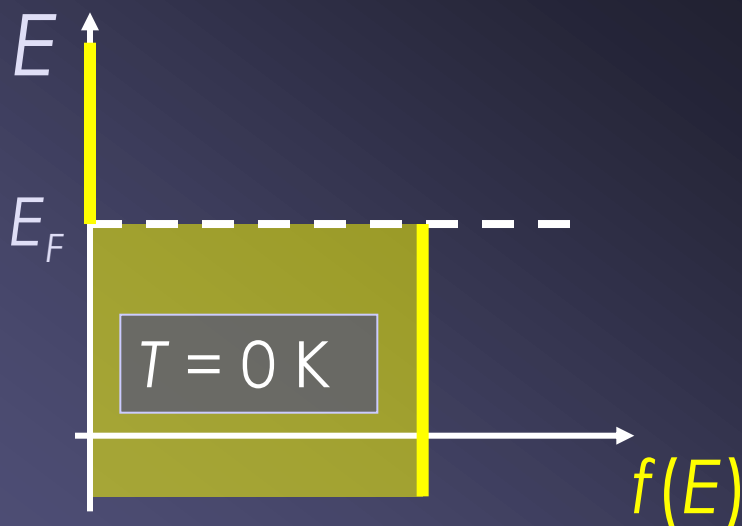
The Fermi level is the highest energy level occupied by electrons:

- Fermi level in the conduction band \implies metal
- Fermi level in the gap \implies insulator

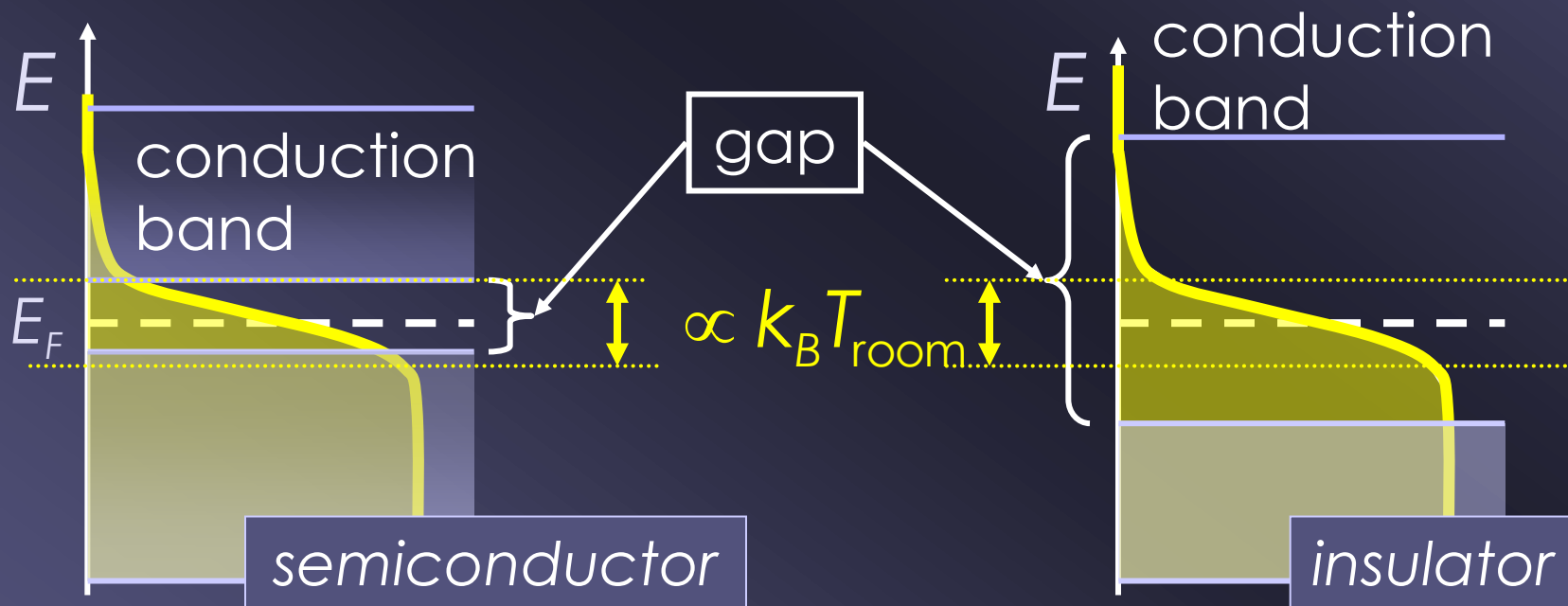


At non-zero temperature, the Fermi level is not strict, the occupation probability will follow Fermi-Dirac statistics

$$f(E) = \frac{1}{1 + \exp \frac{E - E_f}{k_B T}}$$

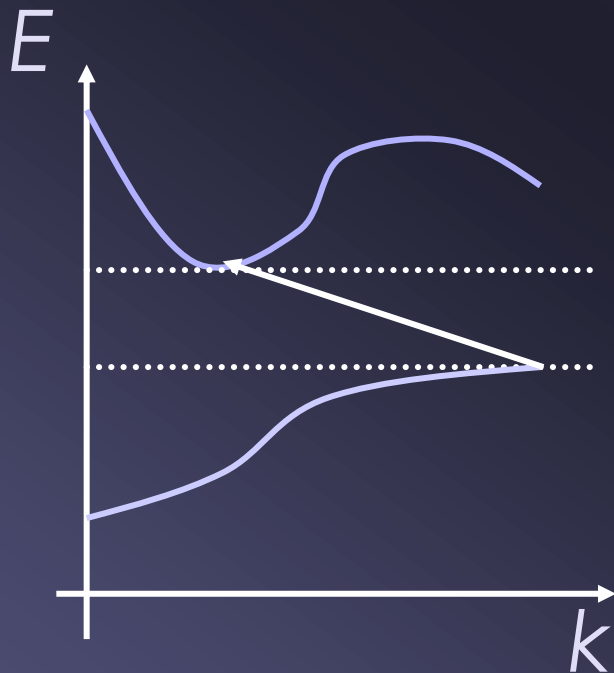


So if an insulator has a bandgap $\propto k_B T_{\text{room}}$, considerable amount of electrons can be present in the conduction band:



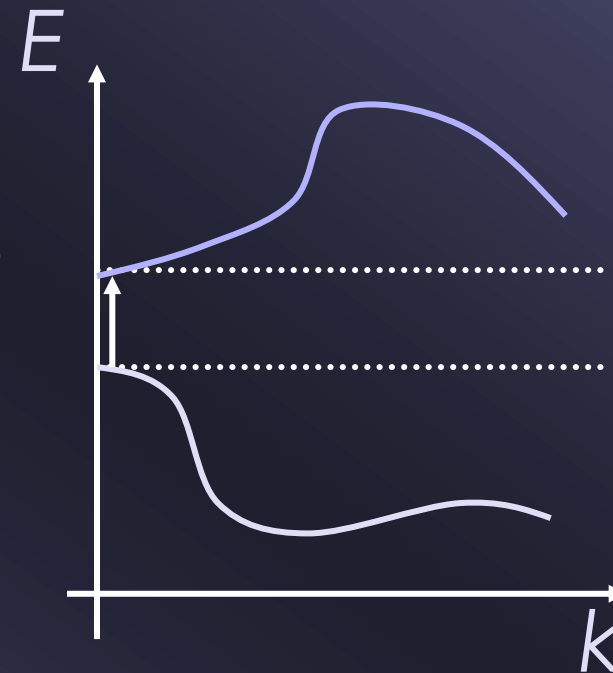
Energy bands in solid states

In a crystal the energy levels depend on the electron's wave number k (quasi momentum):



⋮
 c.b
 indirect
 gap
 v.b
 ⋮

momentum conservation
 ⇒ **no photon emission**

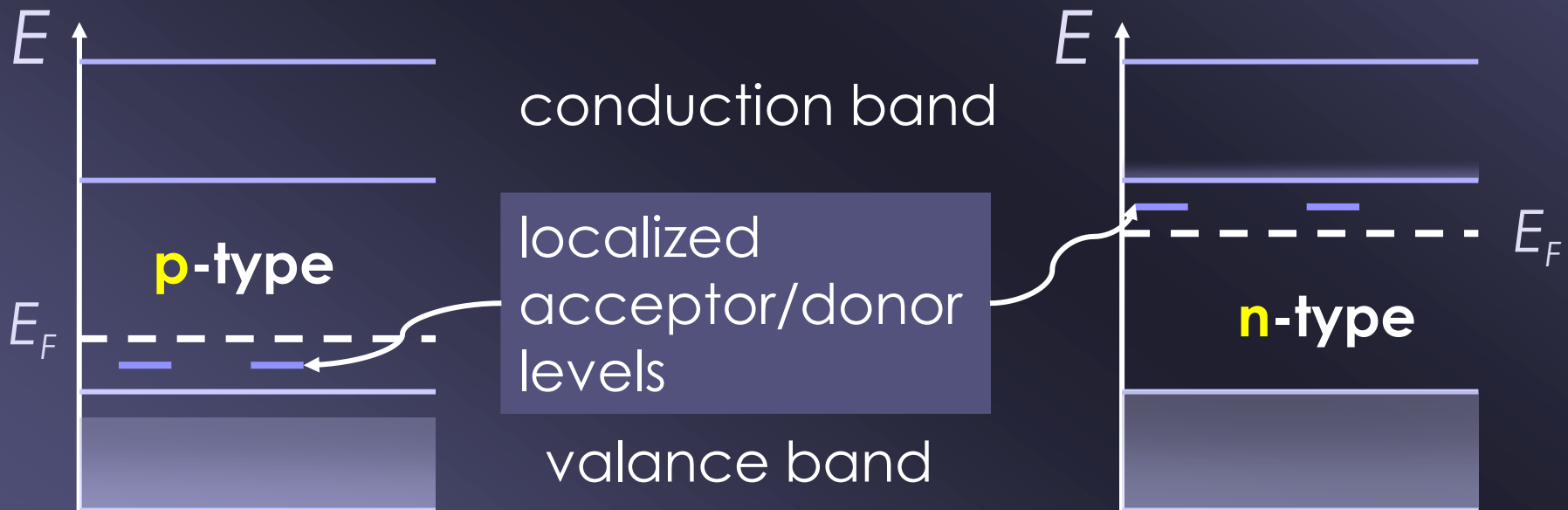


⋮
 c.b
 direct
 gap
 v.b
 ⋮

no momentum to be taken
 ⇒ **photon emission**

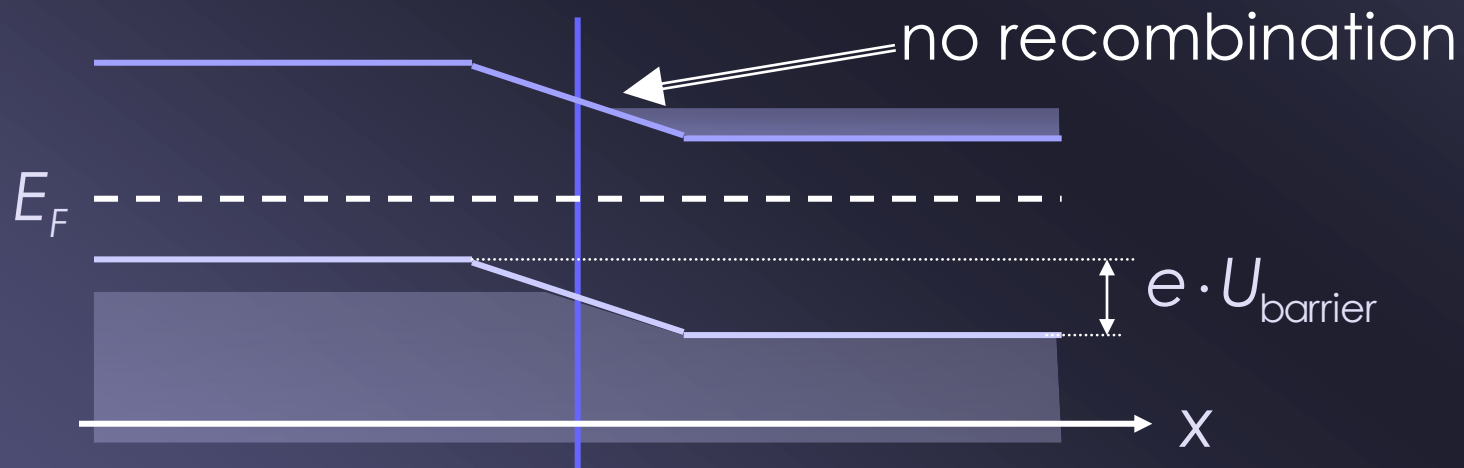
Charge carriers can be injected to semiconductors by doping:

- group V atoms: electrons n-type
- group III atoms: holes p-type



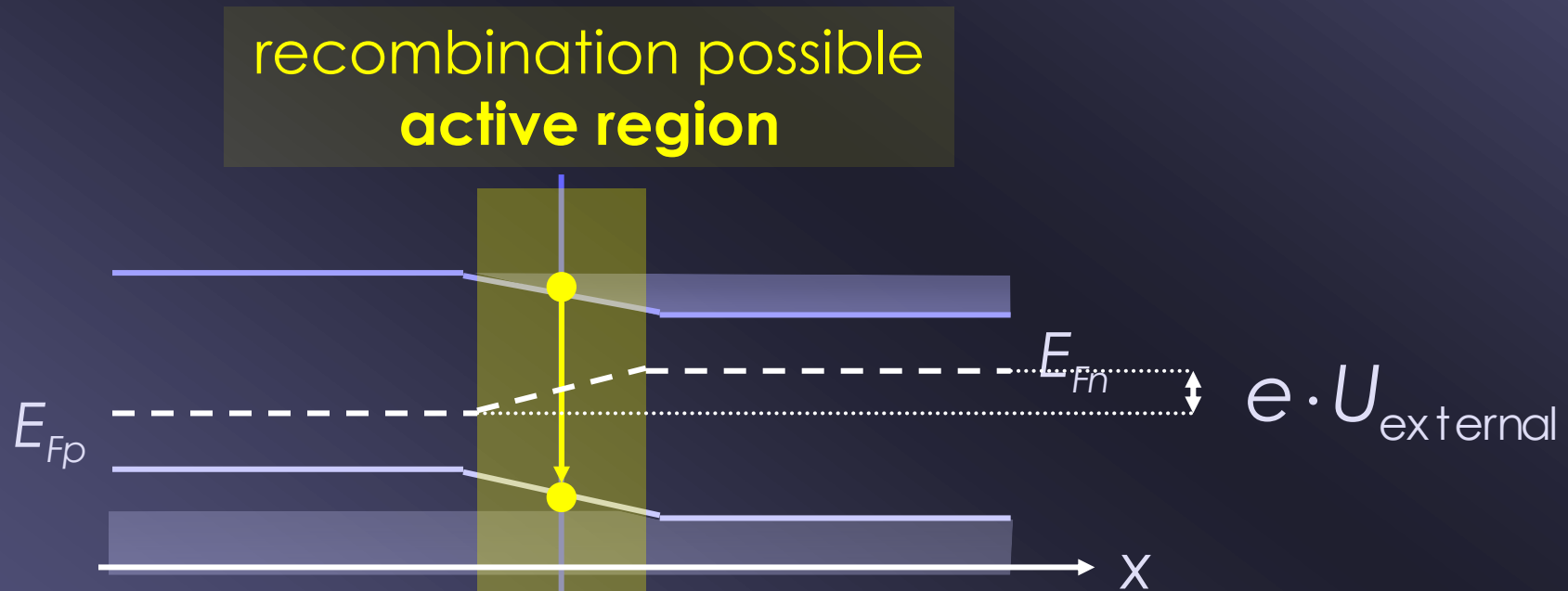
If n-type and p-type doped semiconductor layers are brought in contact,

- the positive and negative charge carriers near the junction can recombine
- photons can be emitted
- potential barrier builds



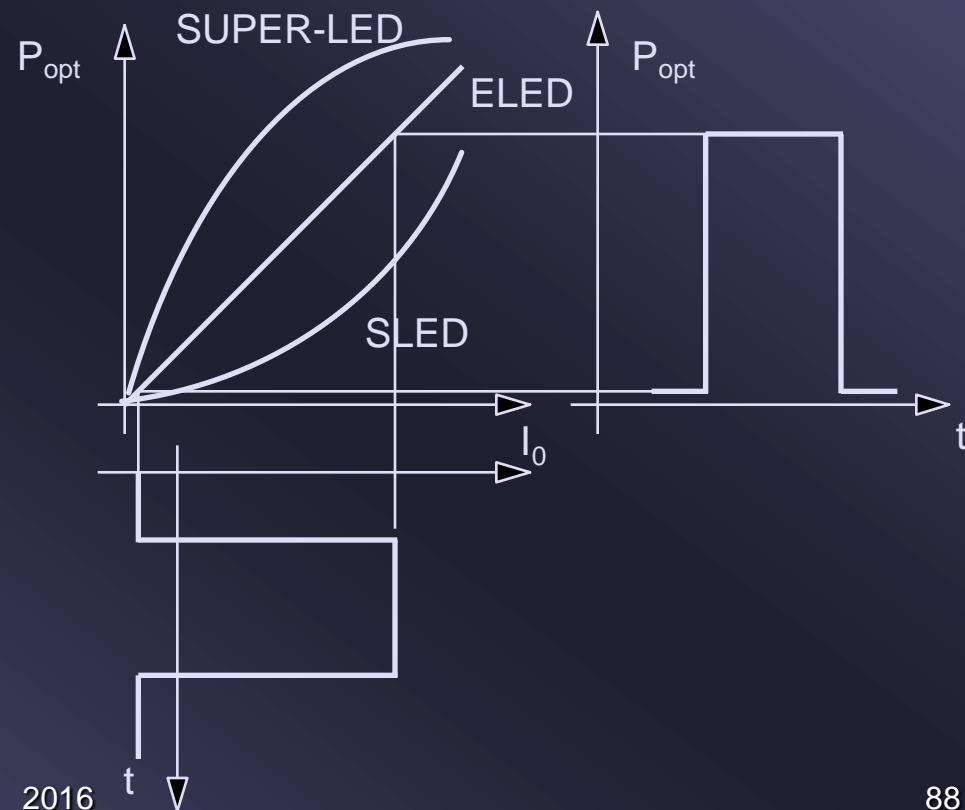
If n-type and p-type doped semiconductor layers are brought in contact,

- the recombination stops, unless external bias is applied \implies LEDs



If n-type and p-type doped semiconductor layers are brought in contact,

- the recombination stops, unless external bias is applied LEDs
- 1 ns – 100 ns



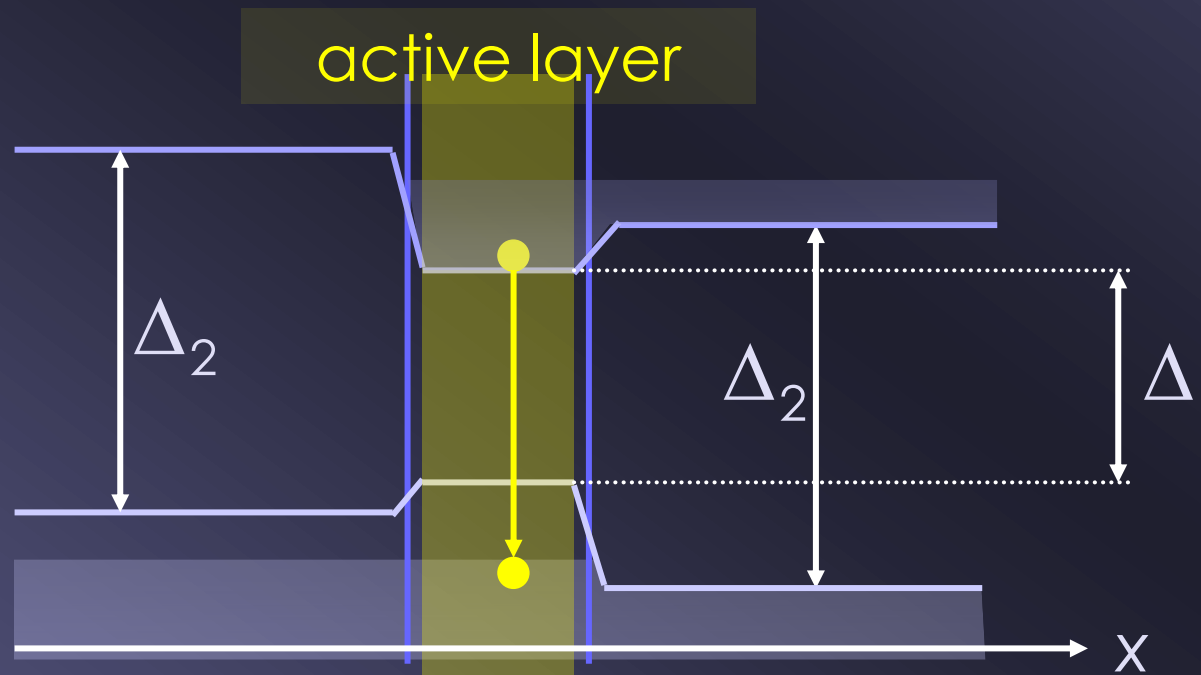
The simple heterojunctions have some disadvantages

- due to the relative large spatial dimension, high current is needed for creating sufficient population inversion
- the heat generated by the current is very high, destroys the device

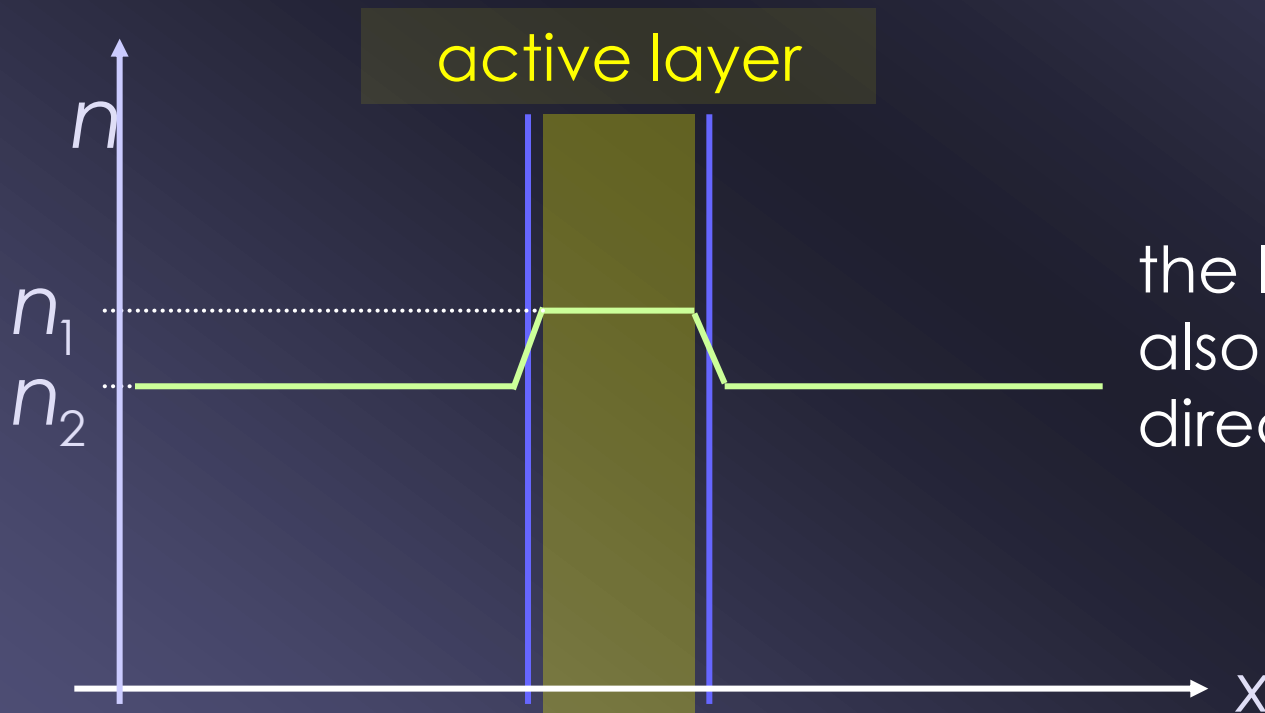
Solution:

restrict the high current density region into small region \implies **double heterojunction**

The double heterojunction localizes the population inversion into a small region of space applying two different materials with different bandgaps Δ_1 and Δ_2

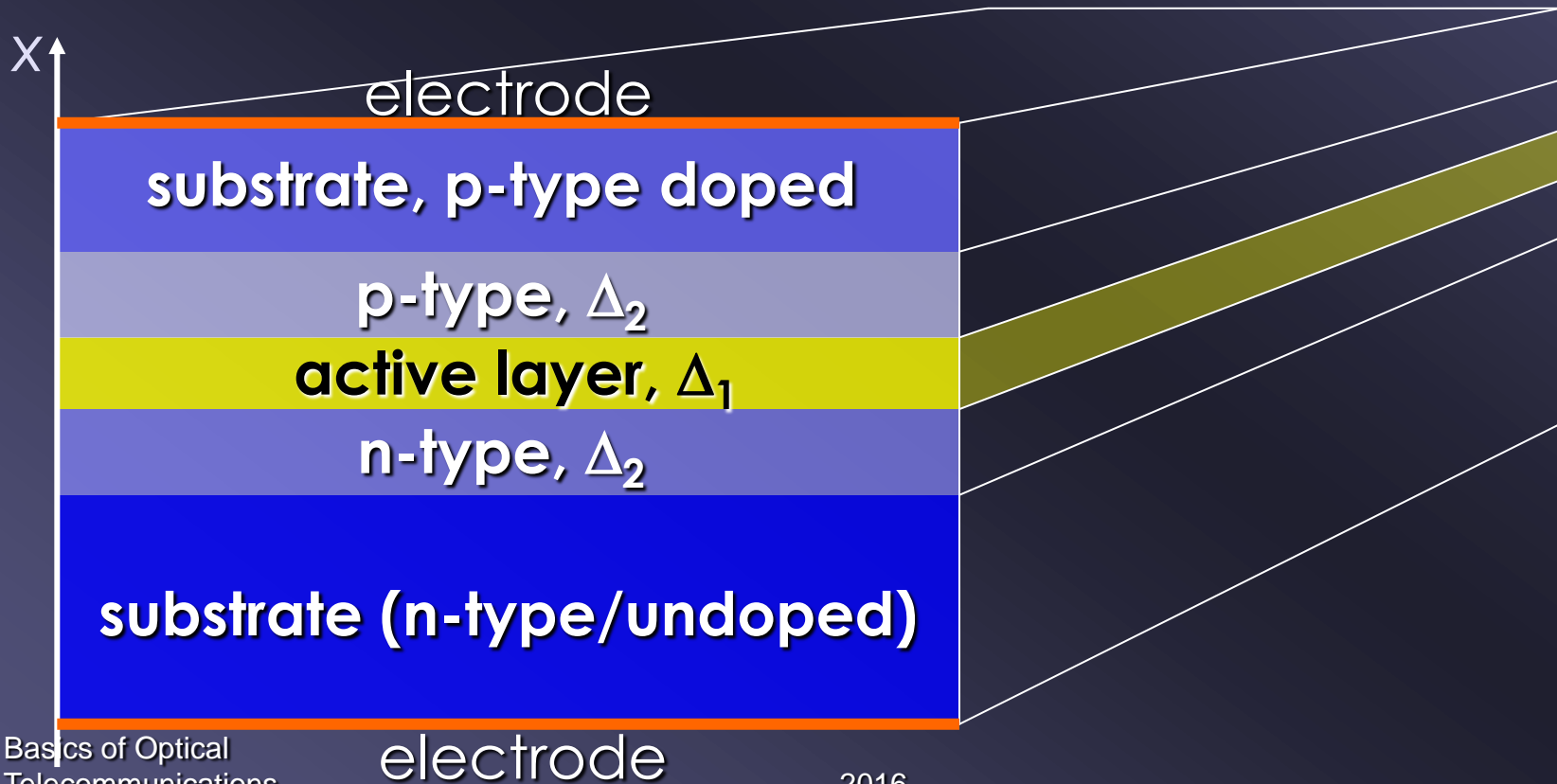


The semiconductors of the double hetero-junction have different refractive indices n_1 and n_2 (not just different bandgaps Δ_1, Δ_2)

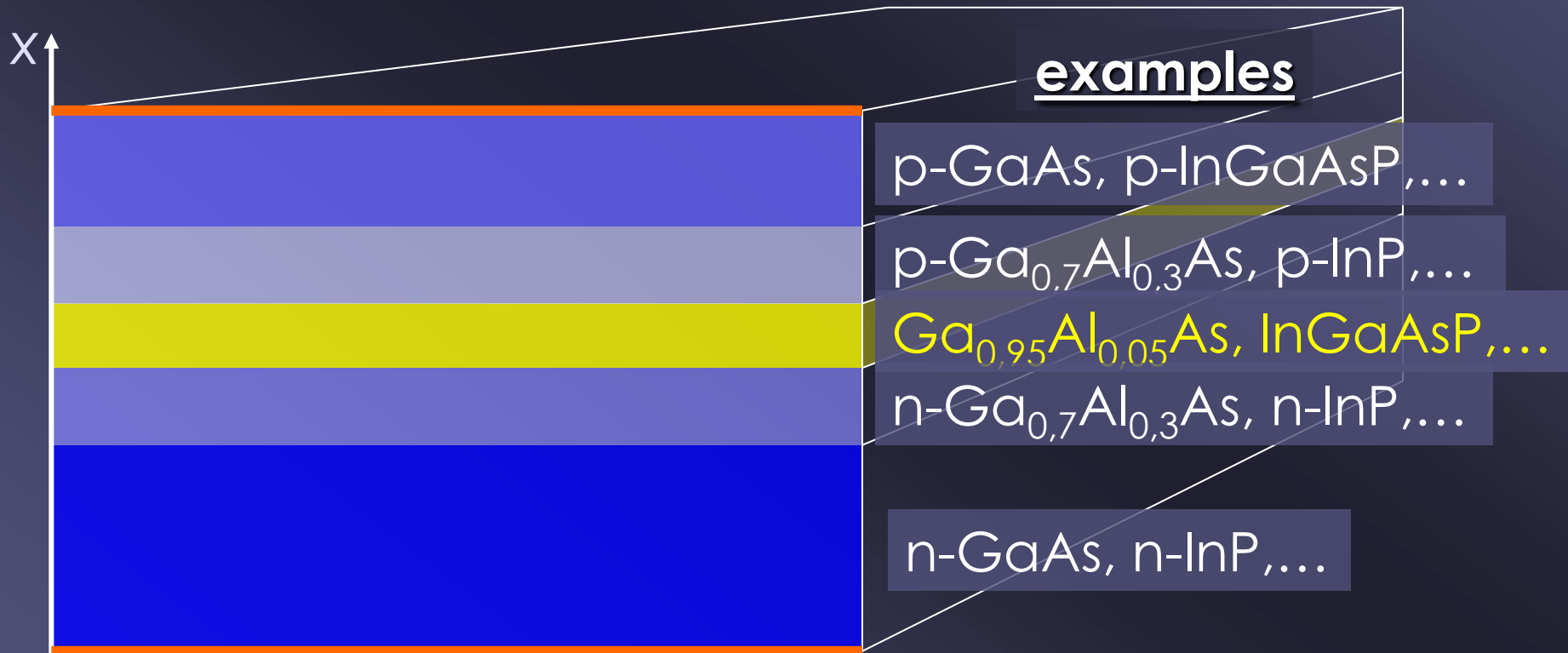


the laser beam is also localized in direction x

The double heterojunction localizes the population inversion and the laser beam into a small region of space \longrightarrow less heat

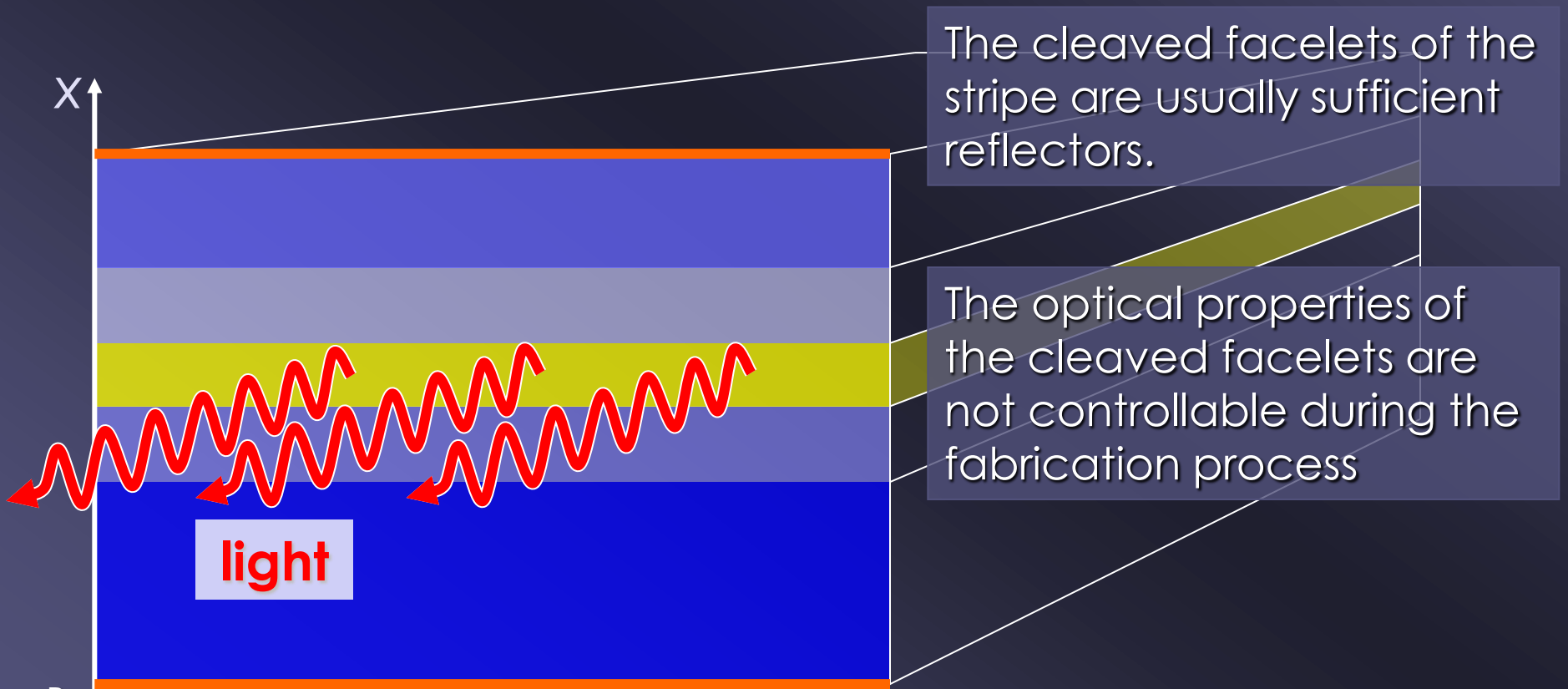


Materials grown upon each other should have similar grid distance in order not to produce strain or dislocations in the crystal.

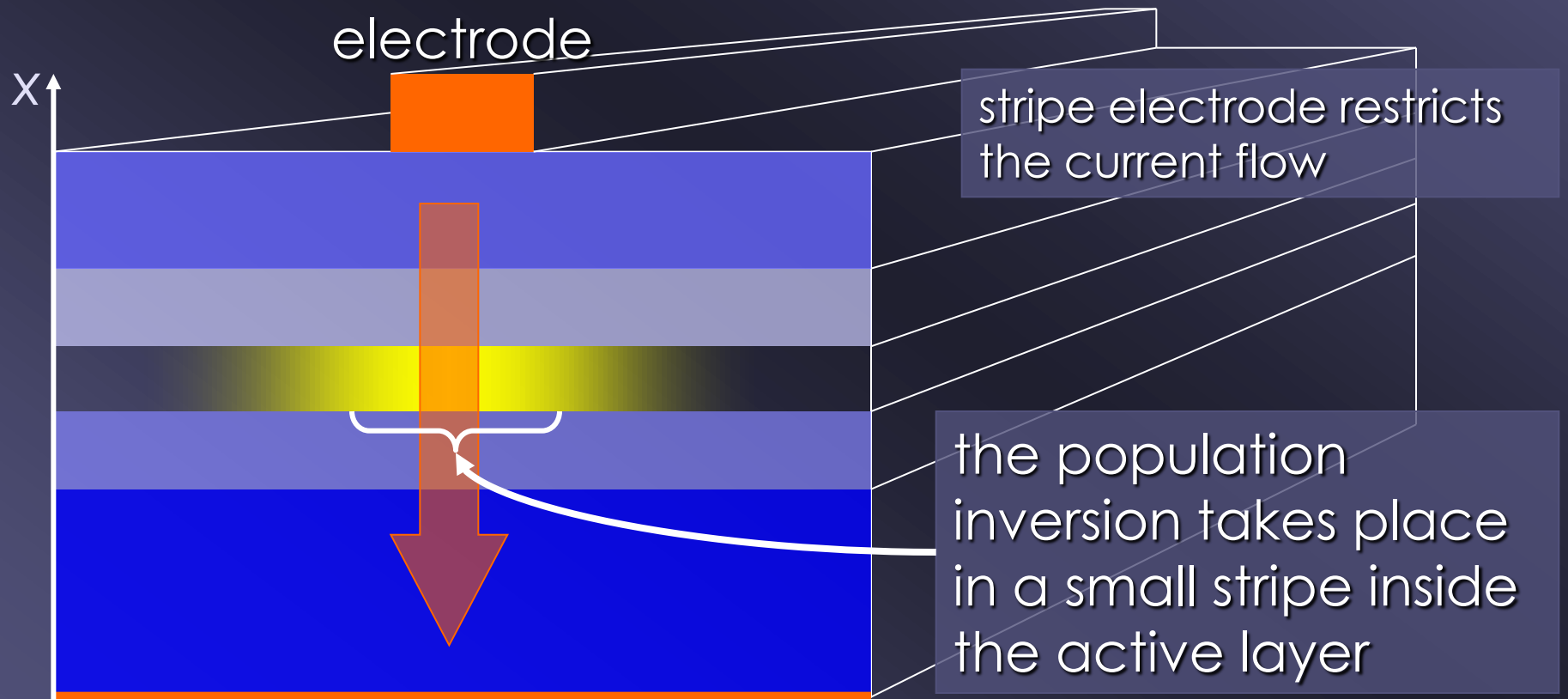


- Thin layers of semiconductors have to be grown on each other with very accurate layer thickness:
- metal-organic chemical vapor deposition
 - molecular beam epitaxy

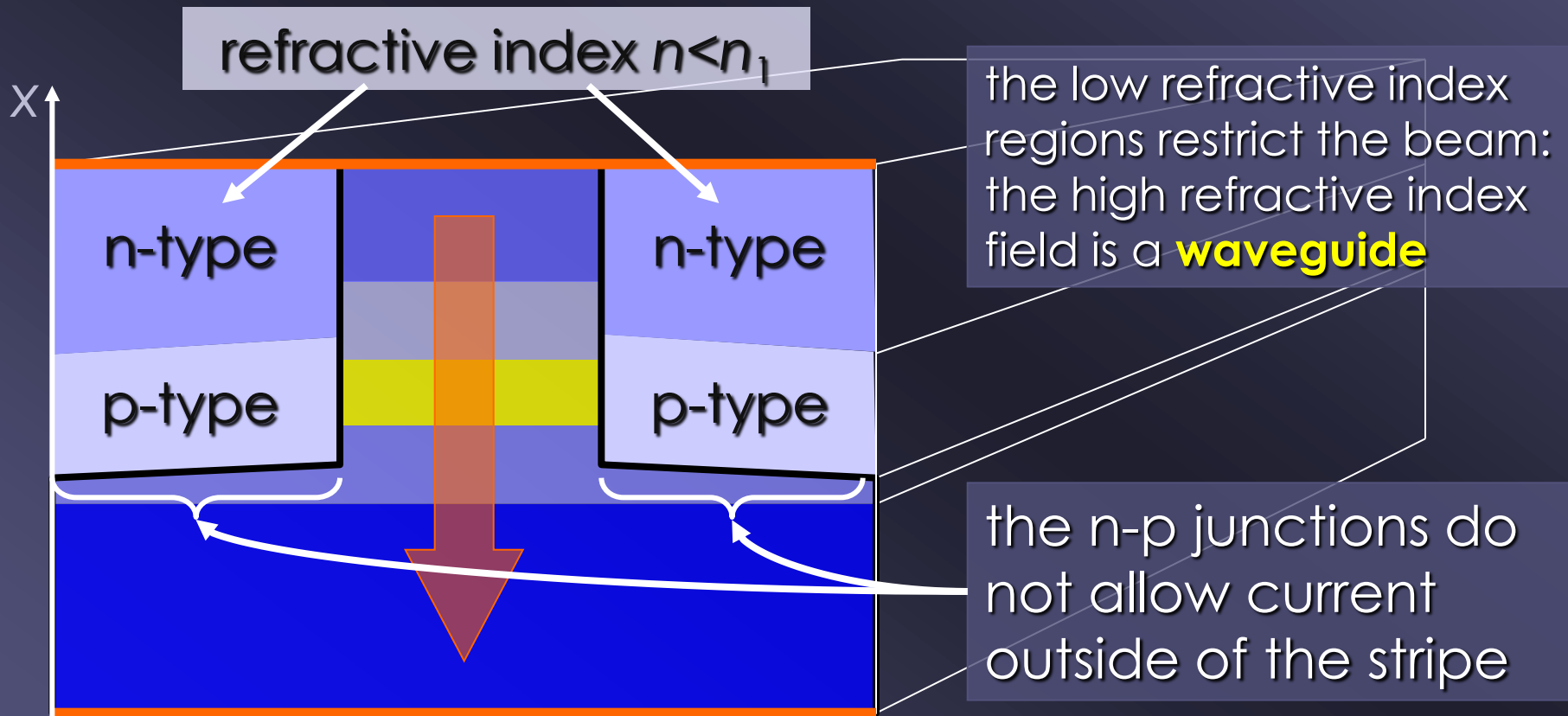
The mirrors placed parallel to the plane plotted the light propagates parallel to the layer



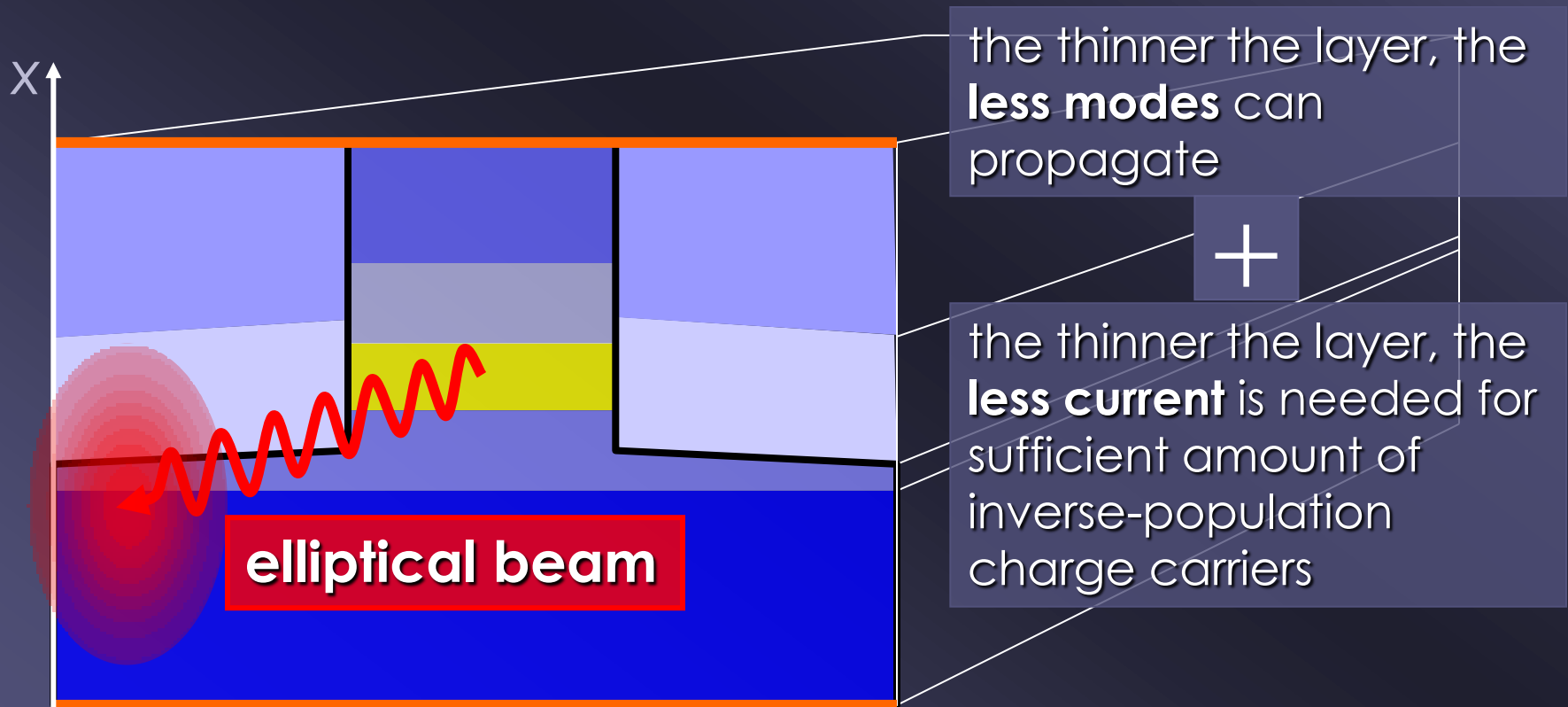
The population inversion can be restricted in the other dimension, too:



With special geometry the laser beam can be localized, as well as the population inversion



With special geometry the laser beam can be localized, as well as the population inversion



For proper optical confinement single waveguide mode is needed \implies the higher order modes have to be cut off.

This requires thickness

$$d \propto \frac{\lambda}{2\sqrt{n_g^2 - n_c^2}}$$

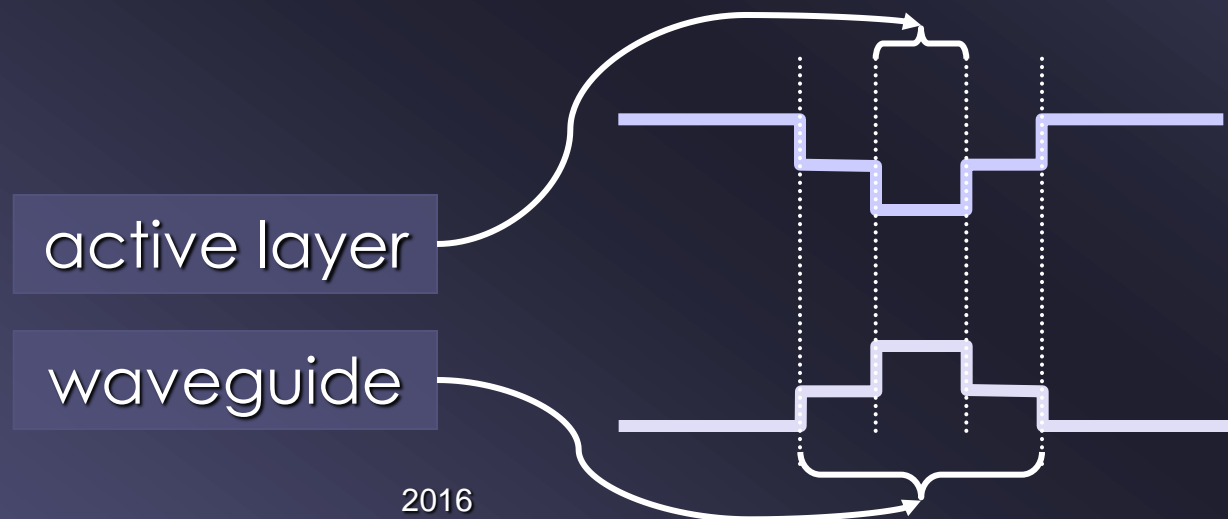
or less. For $\lambda = 1.3 \mu\text{m}$, $d < 0.56 \mu\text{m}$.

(n_g and n_c are refractive indices of waveguide and the cladding)

If the waveguide is too thin, the light spreads out of it
→ the loss increases.

For confining the population inversion thinner layers would be needed.

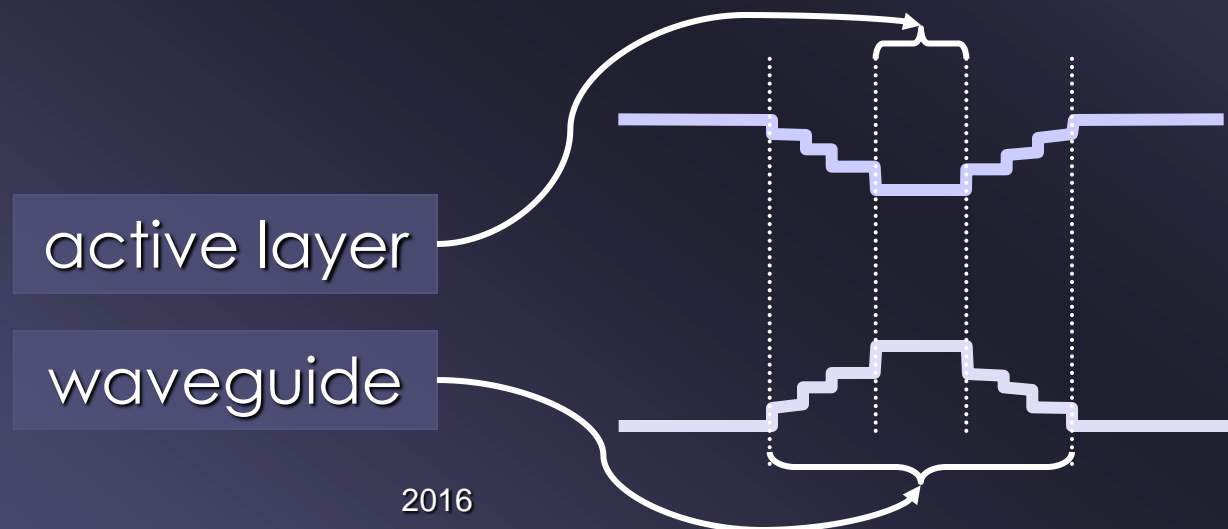
Solution: the waveguide and the active layer are not the same – **S**eparate **C**onfinement **H**eterostructure (SCH)



If the waveguide is too thin, the light spreads out of it
→ the loss increases.

For confining the population inversion thinner layers would be needed.

Solution: the waveguide and the active layer are not the same – **GR**aded **IN**dex SCH (GRINSCH)



If the active region is thin enough, ~10 nm

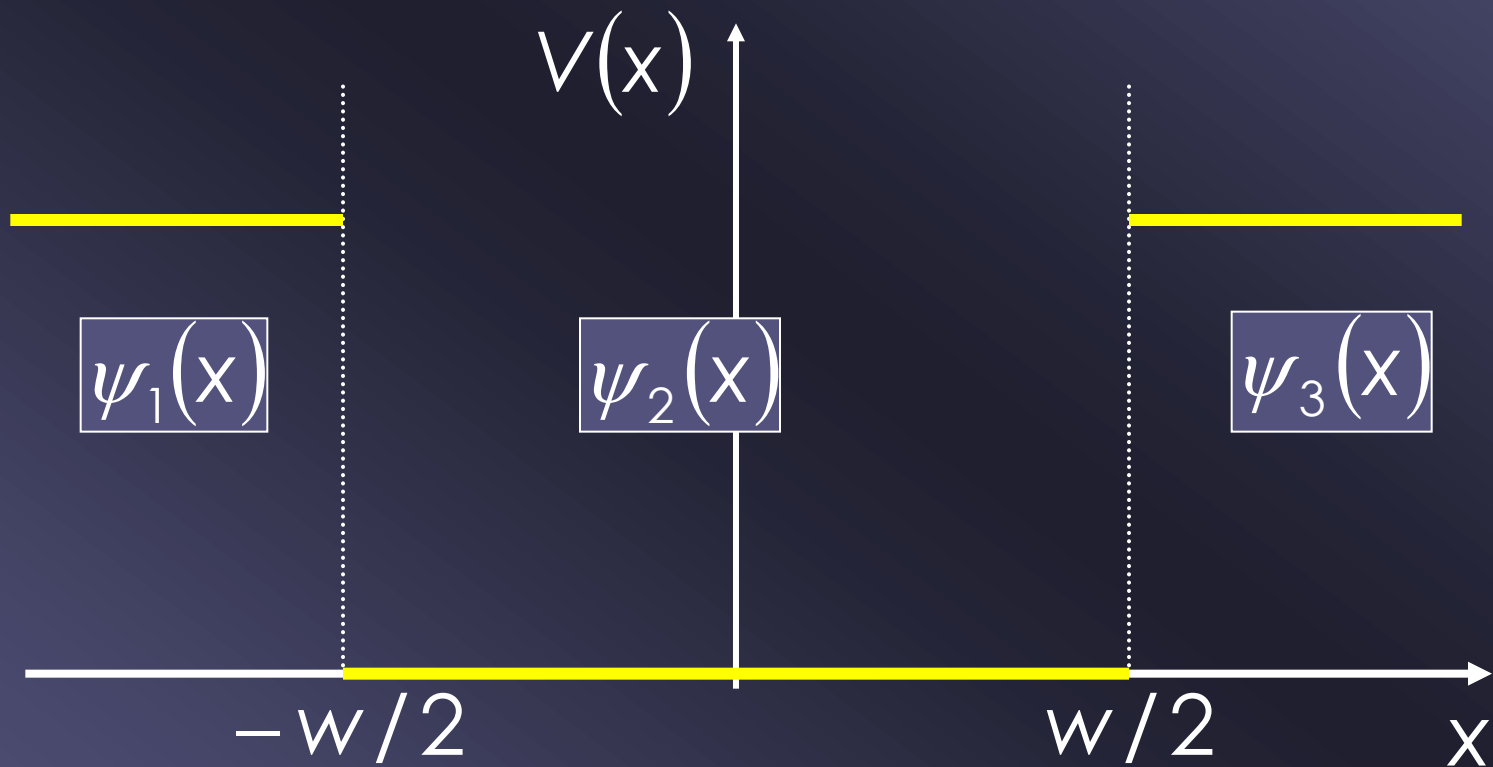
- only few layers of atoms in the active region
- **quantum well** is formed

The solution of the Schrödinger equation of quantum wells:

- I. electron in a potential well in the x direction
- II. free electron gas solution in the yz plane

$$E(\mathbf{k}) = \frac{\hbar^2(k_y^2 + k_z^2)}{2m}$$

The solution of the 1D potential well problem:



The solution of the 1D potential well problem:

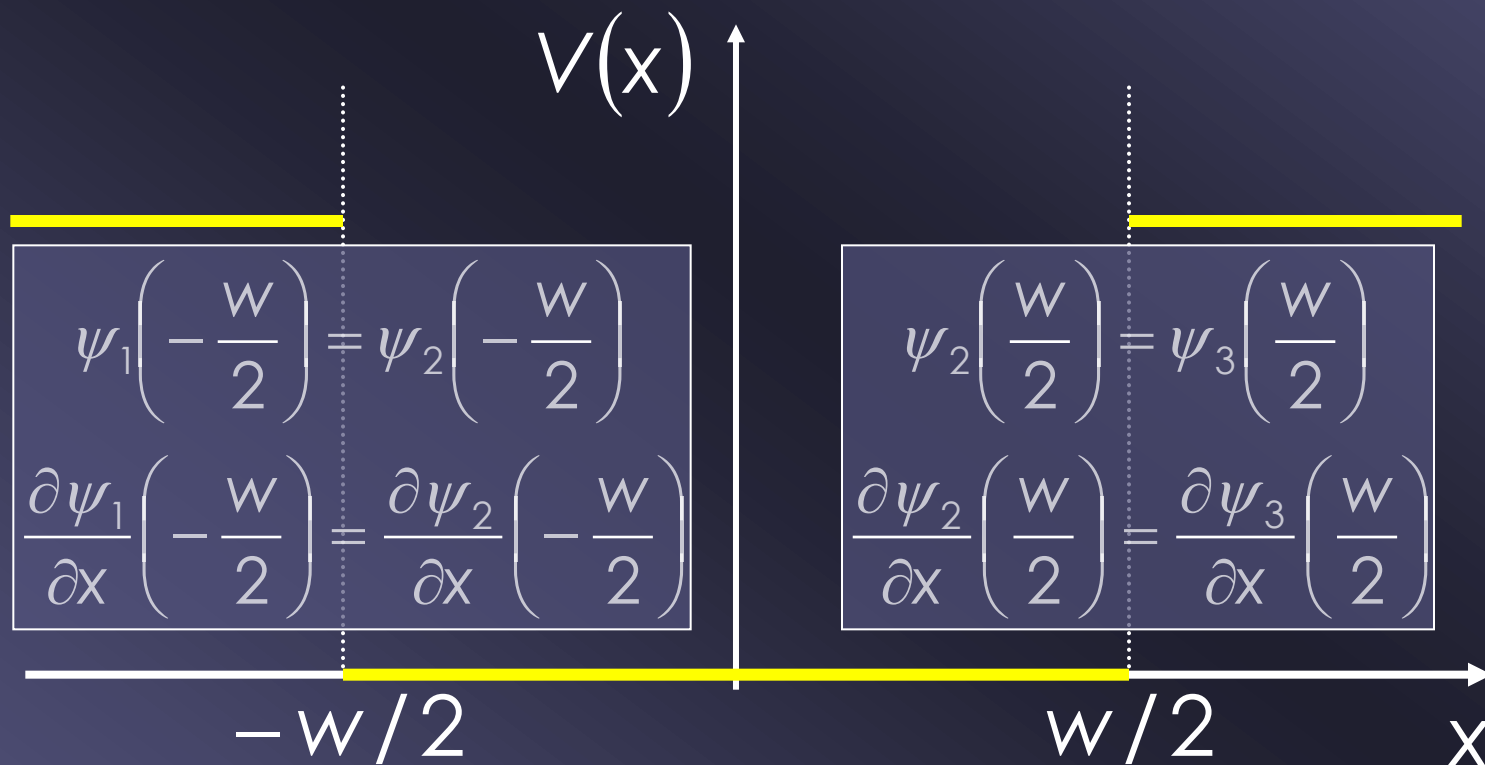
- the Schrödinger equation

$$\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi_1(x) + V_0 \psi_1(x) = E \psi_1(x) \quad x < -\frac{W}{2}$$

$$\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi_2(x) = E \psi_2(x) \quad -\frac{W}{2} < x < \frac{W}{2}$$

$$\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi_3(x) + V_0 \psi_3(x) = E \psi_3(x) \quad x > \frac{W}{2}$$

- the boundary conditions:



- The solution of the differential equation system:

$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$

$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

with

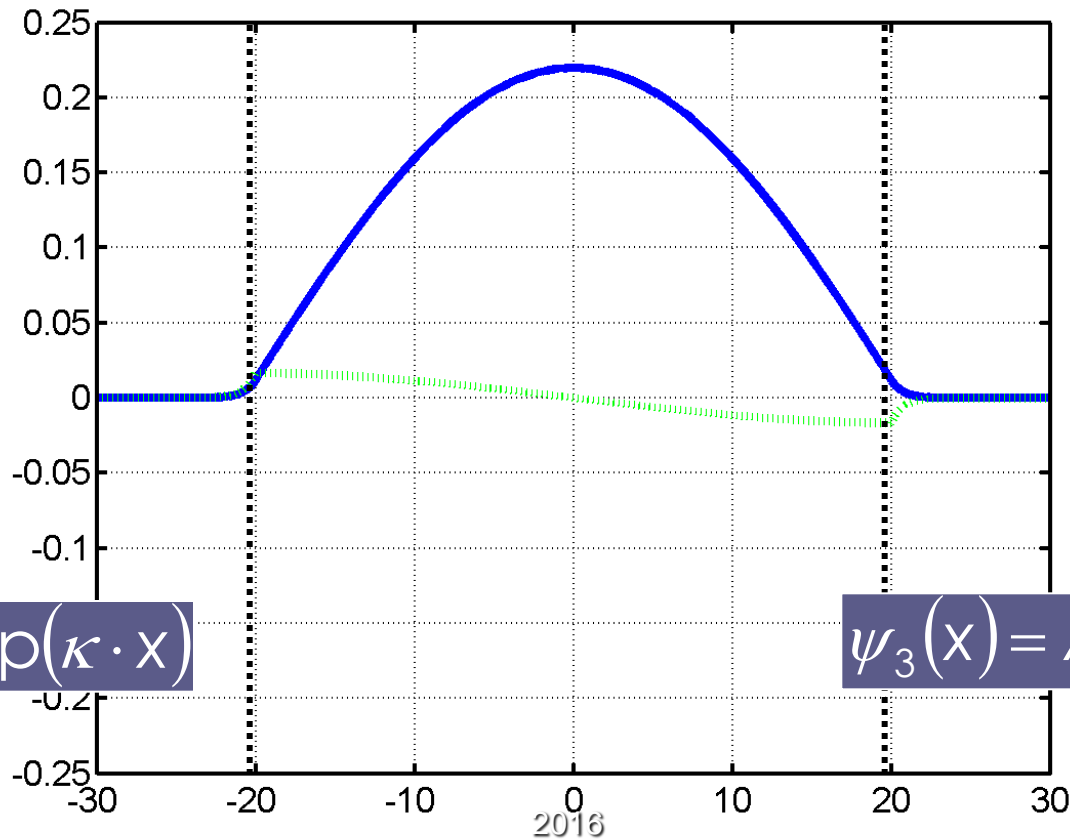
$$\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

and

$$k = \frac{\sqrt{2mE}}{\hbar}$$

- For $V_0=1$ a.u., $w=40$ a.u., $E=0.0029$ a.u.:

$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$

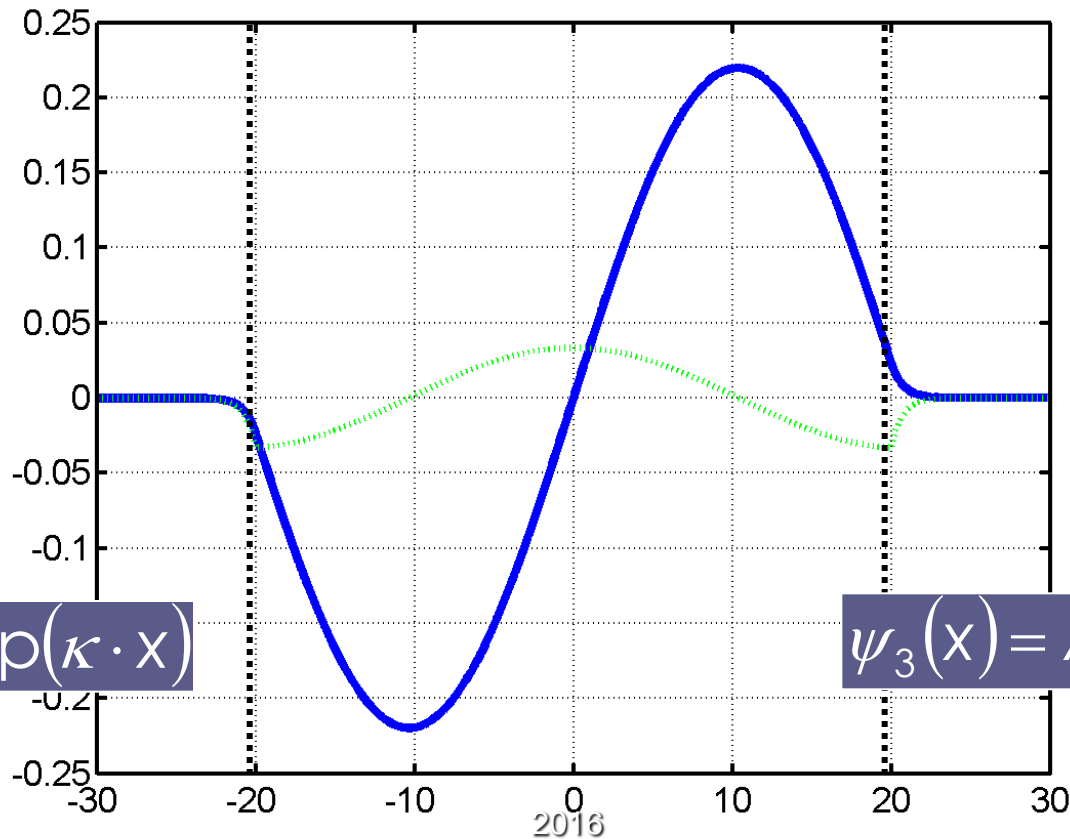


$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

- For $V_0=1$ a.u., $w=40$ a.u., $E=0.0115$ a.u.:

$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$

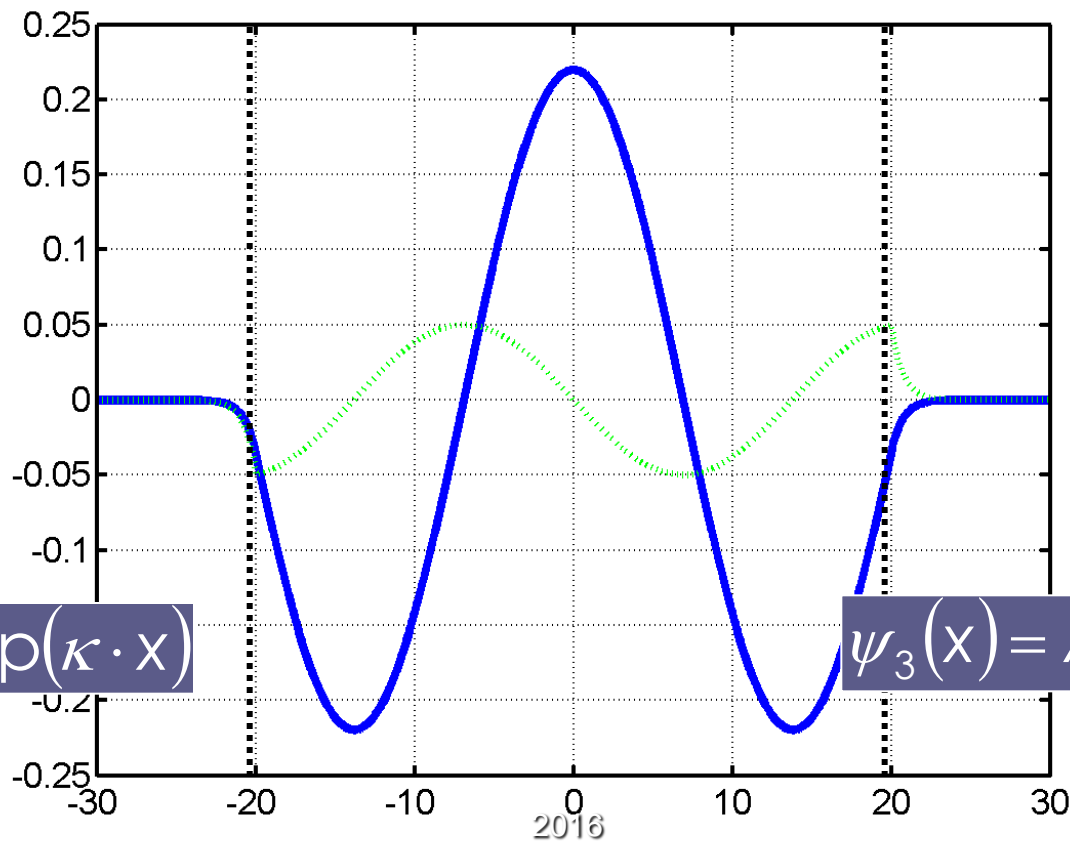


$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

- For $V_0=1$ a.u., $w=40$ a.u., $E=0.0259$ a.u.:

$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$

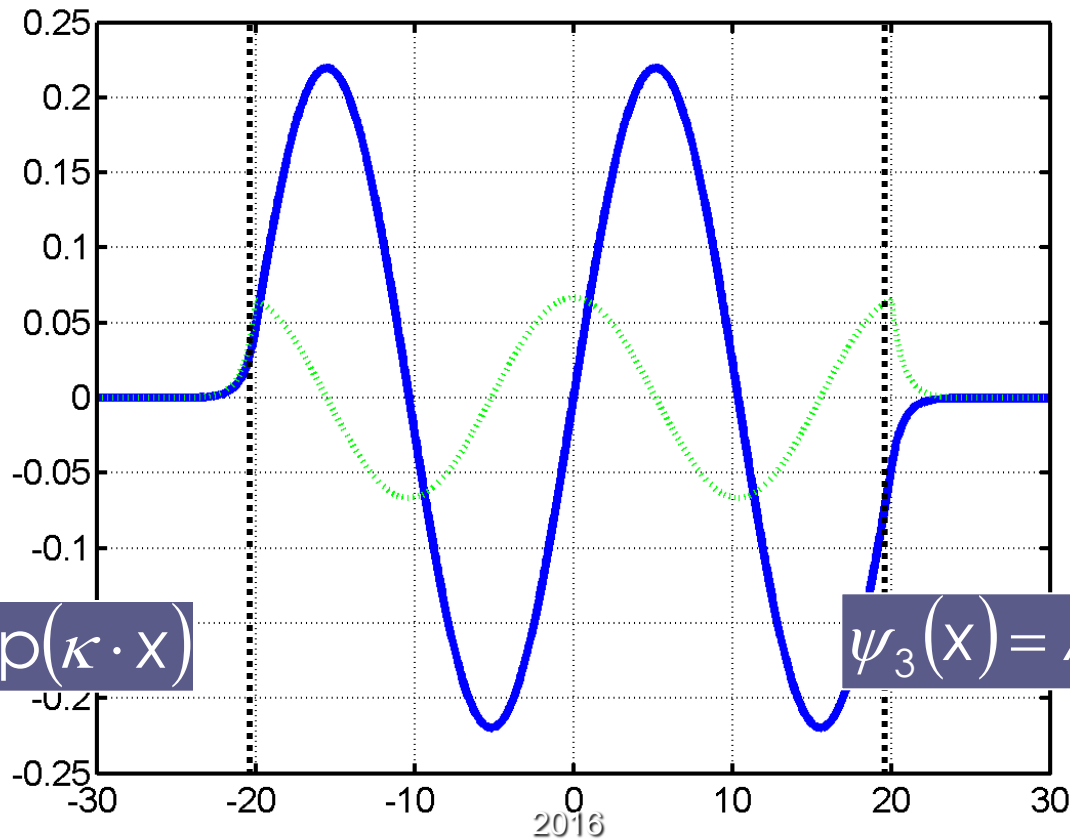


$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

- For $V_0=1$ a.u., $w=40$ a.u., $E=0.0460$ a.u.:

$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$

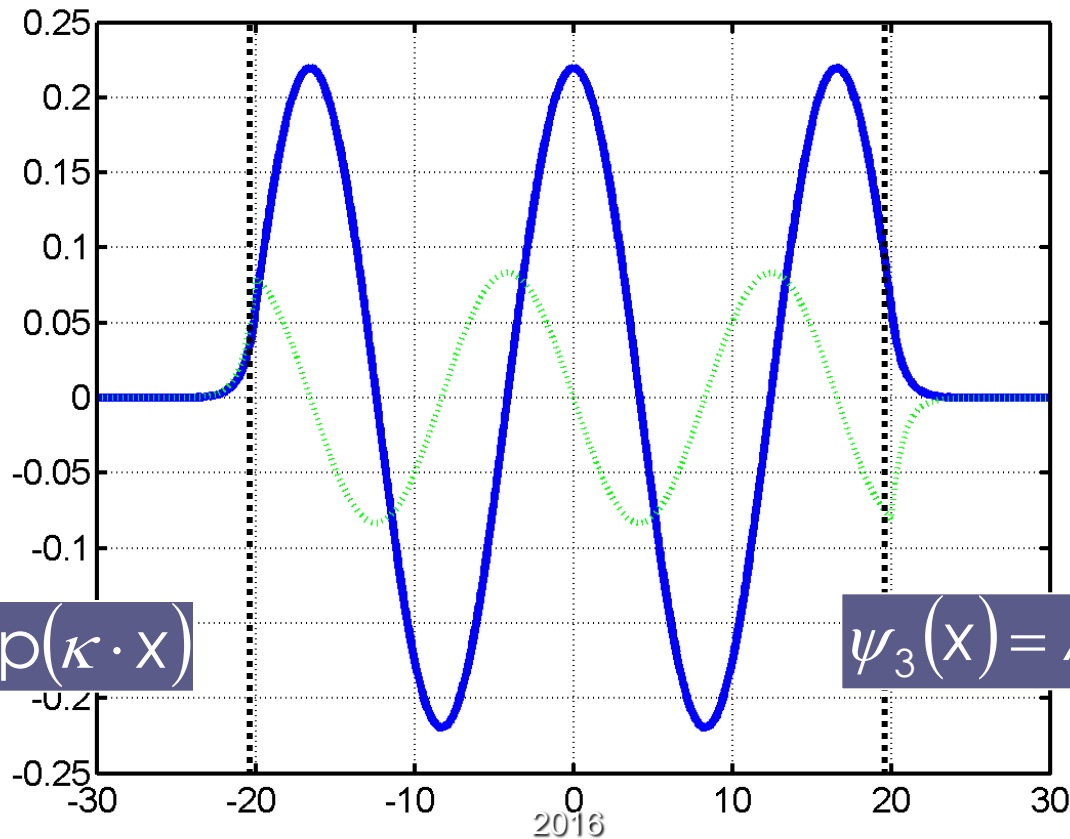


$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

For $V_0=1$ a.u., $w=40$ a.u., $E=0.0718$ a.u.:

$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$

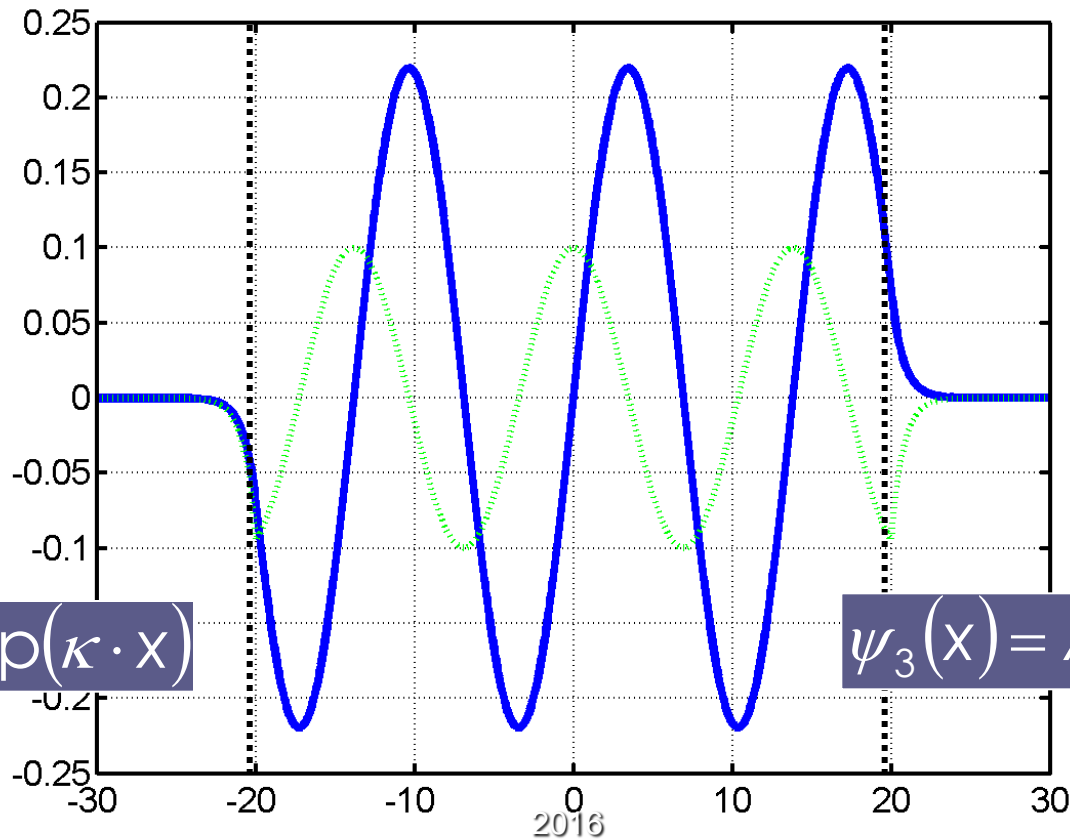


$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

- For $V_0=1$ a.u., $w=40$ a.u., $E=0.1035$ a.u.:

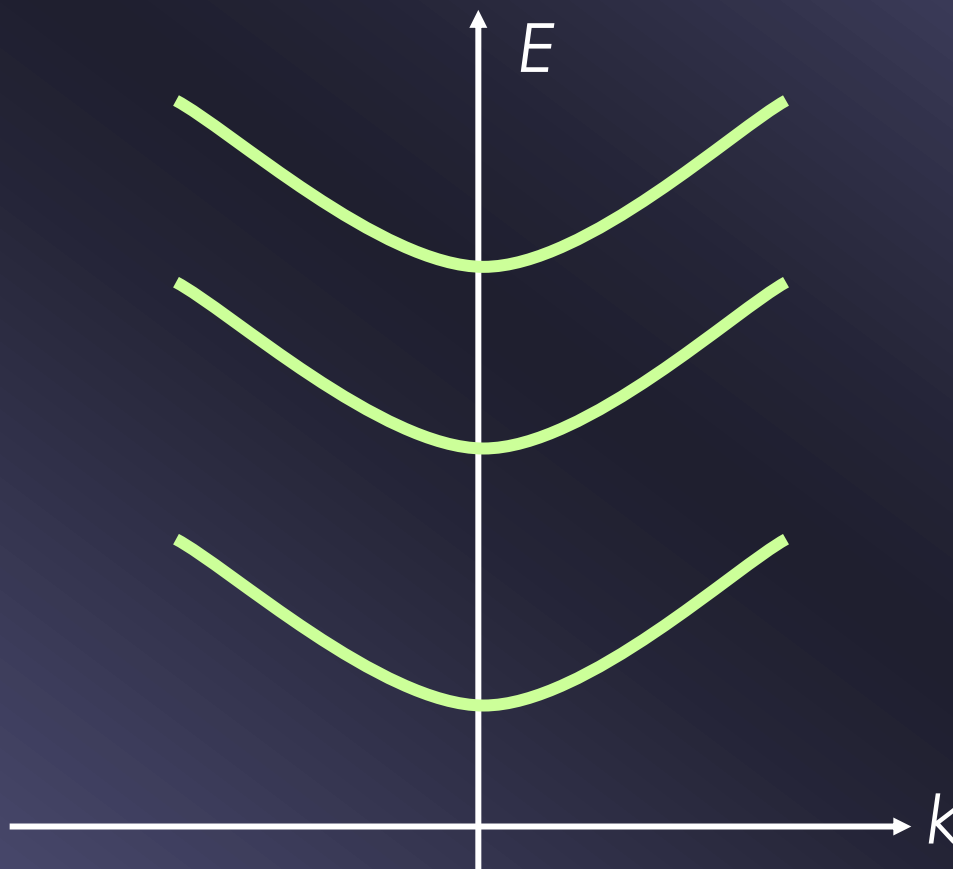
$$\psi_2(x) = a_2 \sin(k \cdot x) + b_2 \cos(k \cdot x)$$



$$\psi_1(x) = A_1 \exp(\kappa \cdot x)$$

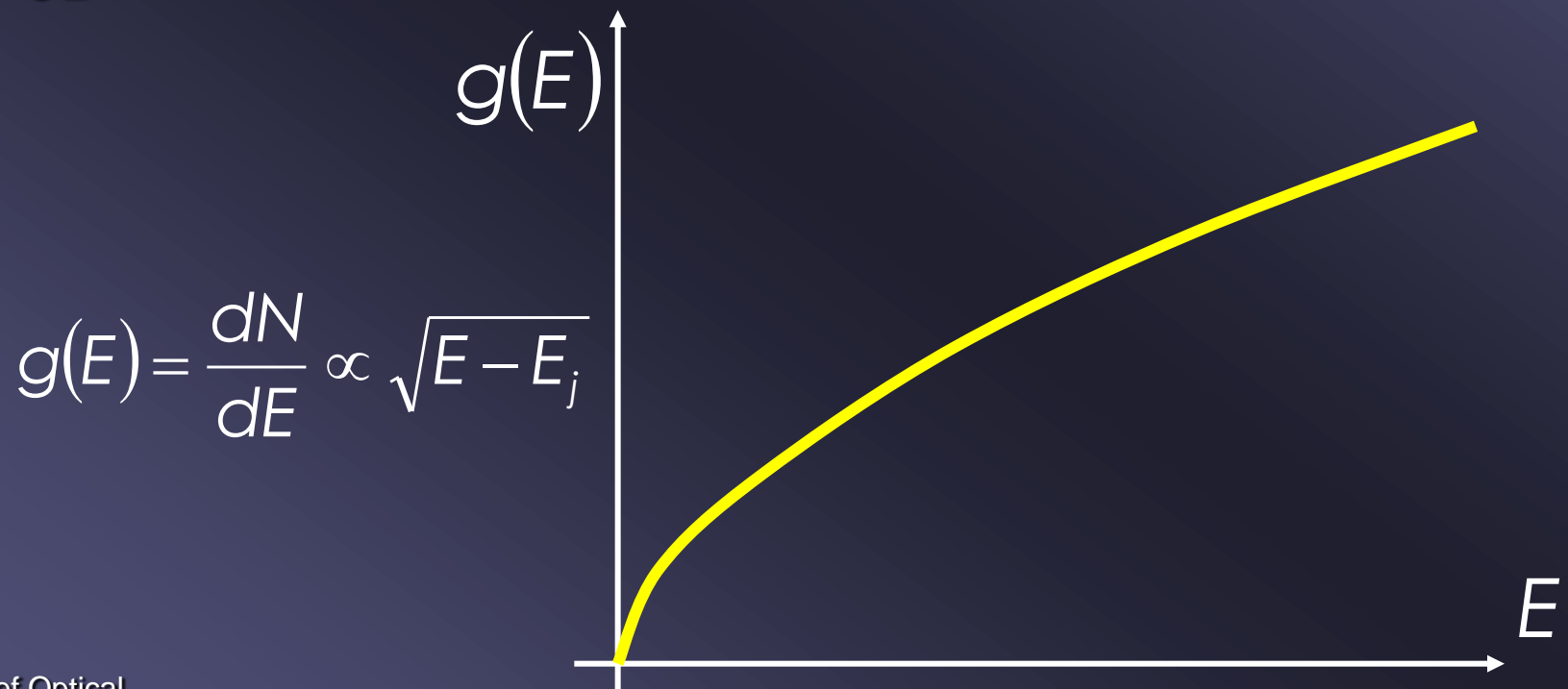
$$\psi_3(x) = A_3 \exp(-\kappa \cdot x)$$

The energy versus quasi momentum function:



If the free electron gas is restricted to two or less dimensions, the density of states behaves different from the 3D case

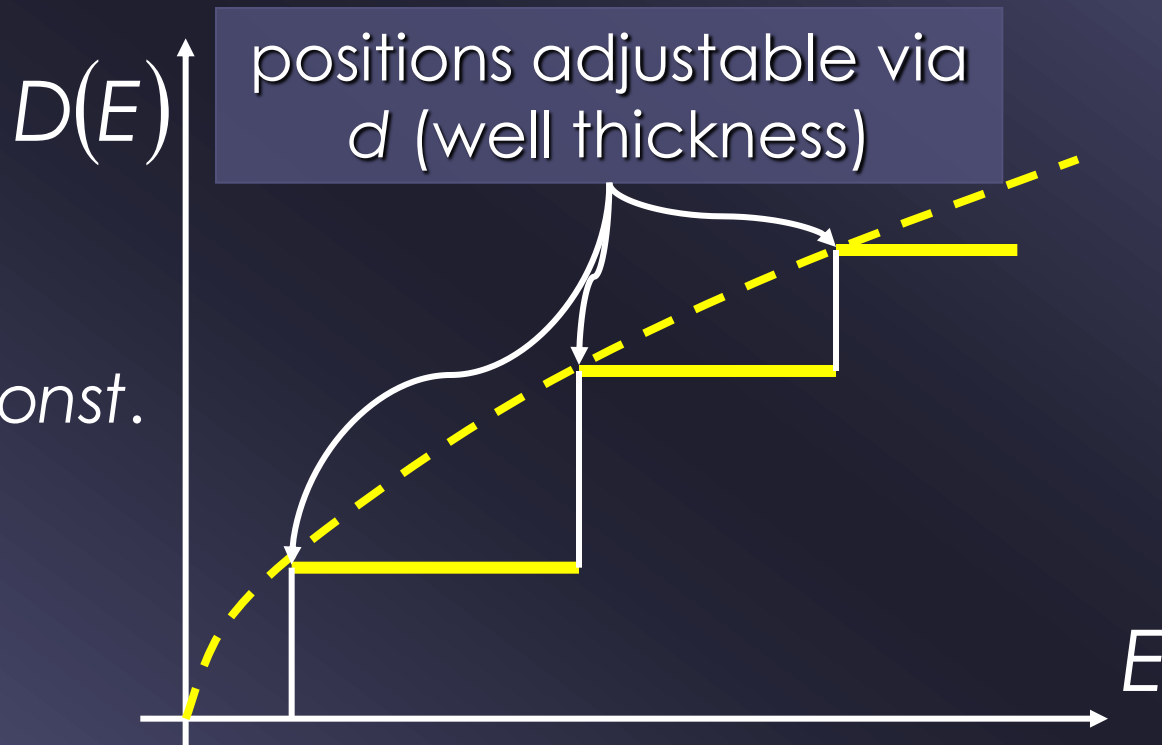
- 3D



If the free electron gas is restricted to two or less dimensions, the density of states behaves different from the 3D case

• 2D

$$D(E) = \frac{dN}{dE} = \text{const.}$$

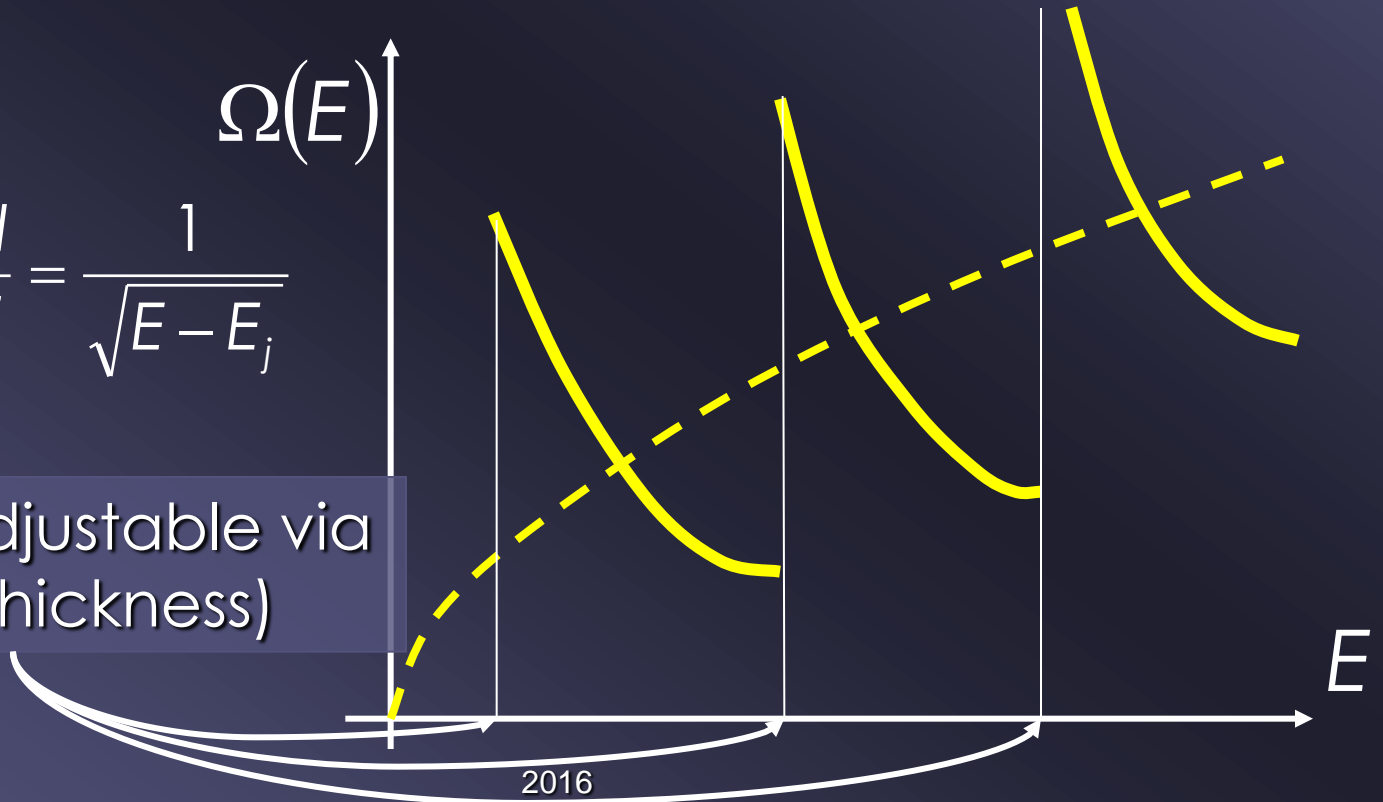


If the free electron gas is restricted to two or less dimensions, the density of states behaves different from the 3D case

● 1D

$$\Omega(E) = \frac{dN}{dE} = \frac{1}{\sqrt{E - E_j}}$$

positions adjustable via d (well thickness)



The absorption spectrum is also different for 2D electron systems from the bulk case:

- 3D:

$$\alpha \propto \sqrt{h\nu - \Delta}$$

- 2D:

the absorption spectrum is steplike with resonances at the frequencies corresponding to the energy differences

better absorption spectrum, transparency.

Usually a single quantum well (SQW) is too thin for confining the light
multiple quantum wells (MQW) with barrier layers can be applied:

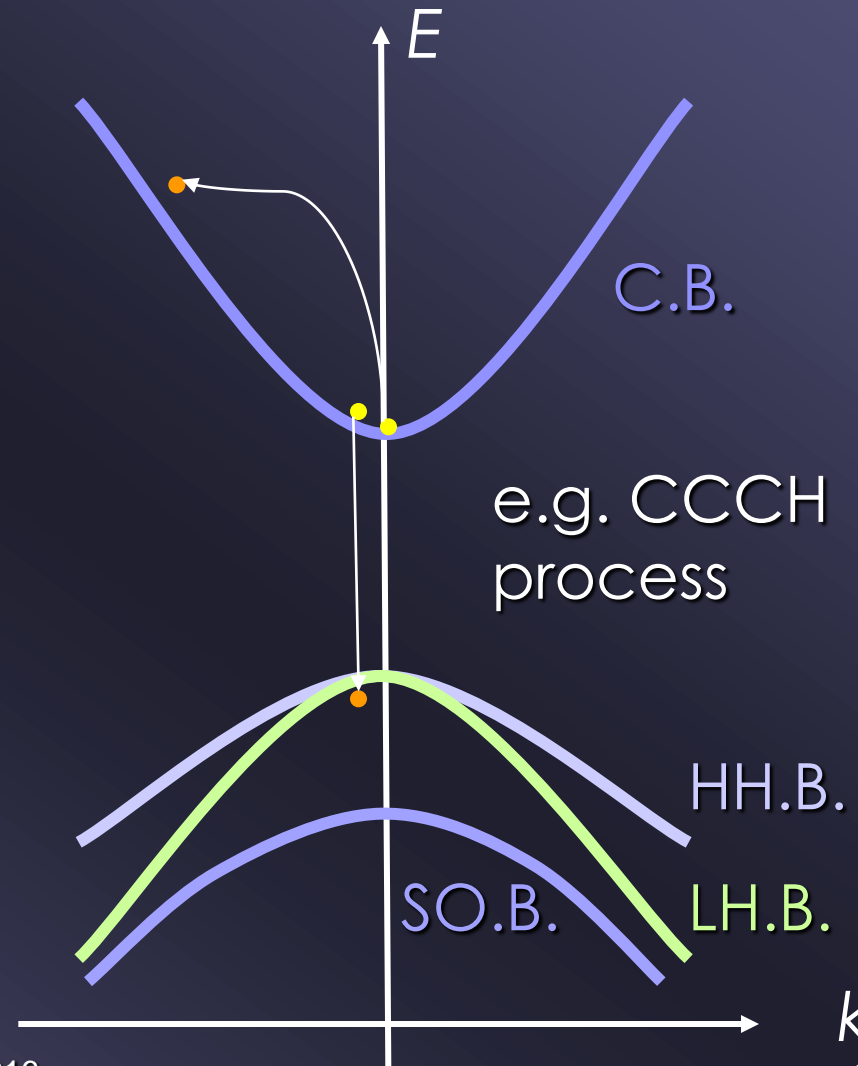


The quantum well lasers have higher threshold than the bulk lasers, but they also have higher gain, better transparency.

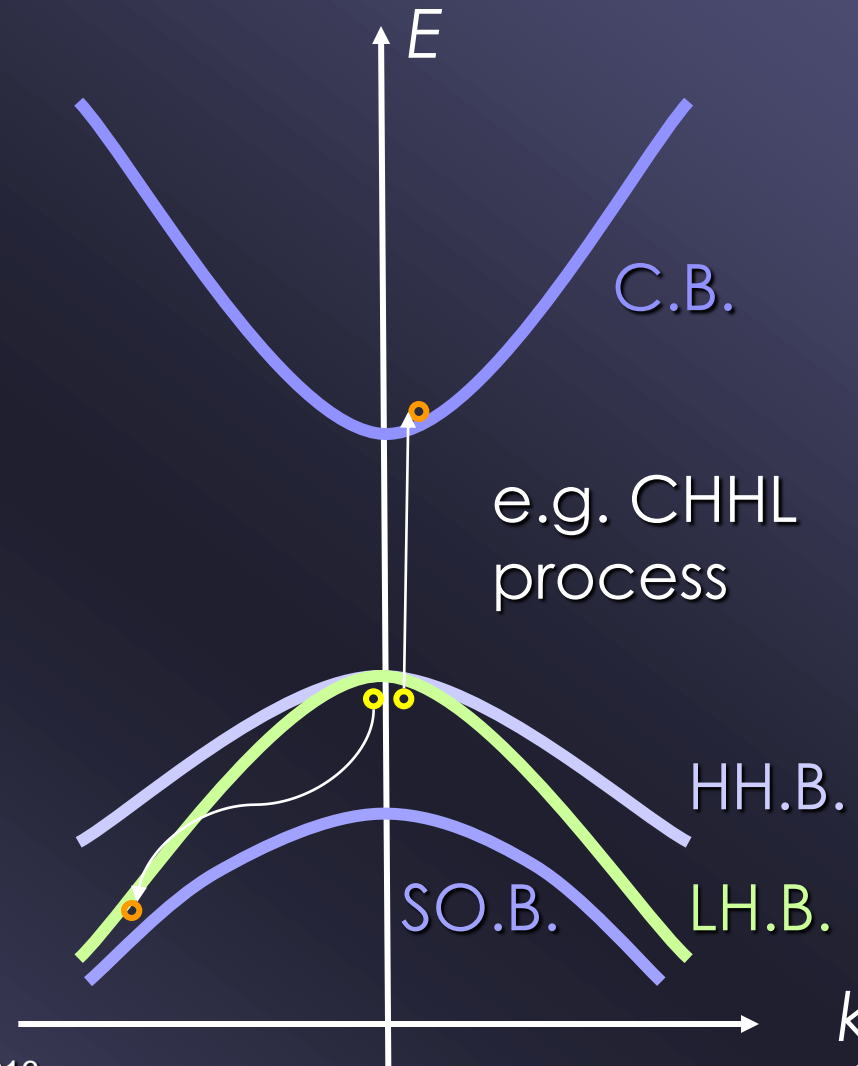
Quantum wells based on GaAs perform well, low loss, high gain

Quantum wells based on InP have higher loss (Auger recombination,...)
a **strain** in the QW layers improves the performance of QW InGaAsP lasers

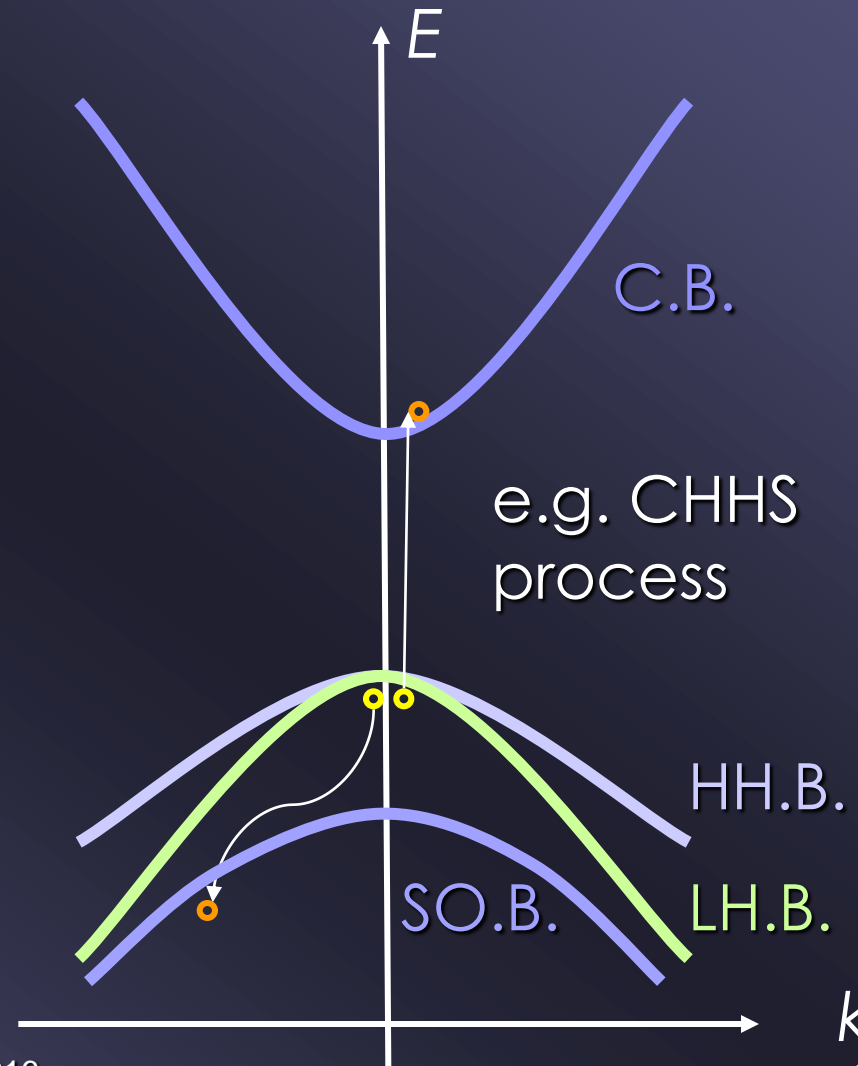
In the Auger recombination the energy which is released via an electron-hole recombination is absorbed by an other electron, which dissipates the energy by generating lattice oscillations (phonons)



In the Auger recombination the energy which is released via an electron-hole recombination is absorbed by an other hole, which dissipates the energy by generating lattice oscillations (phonons)

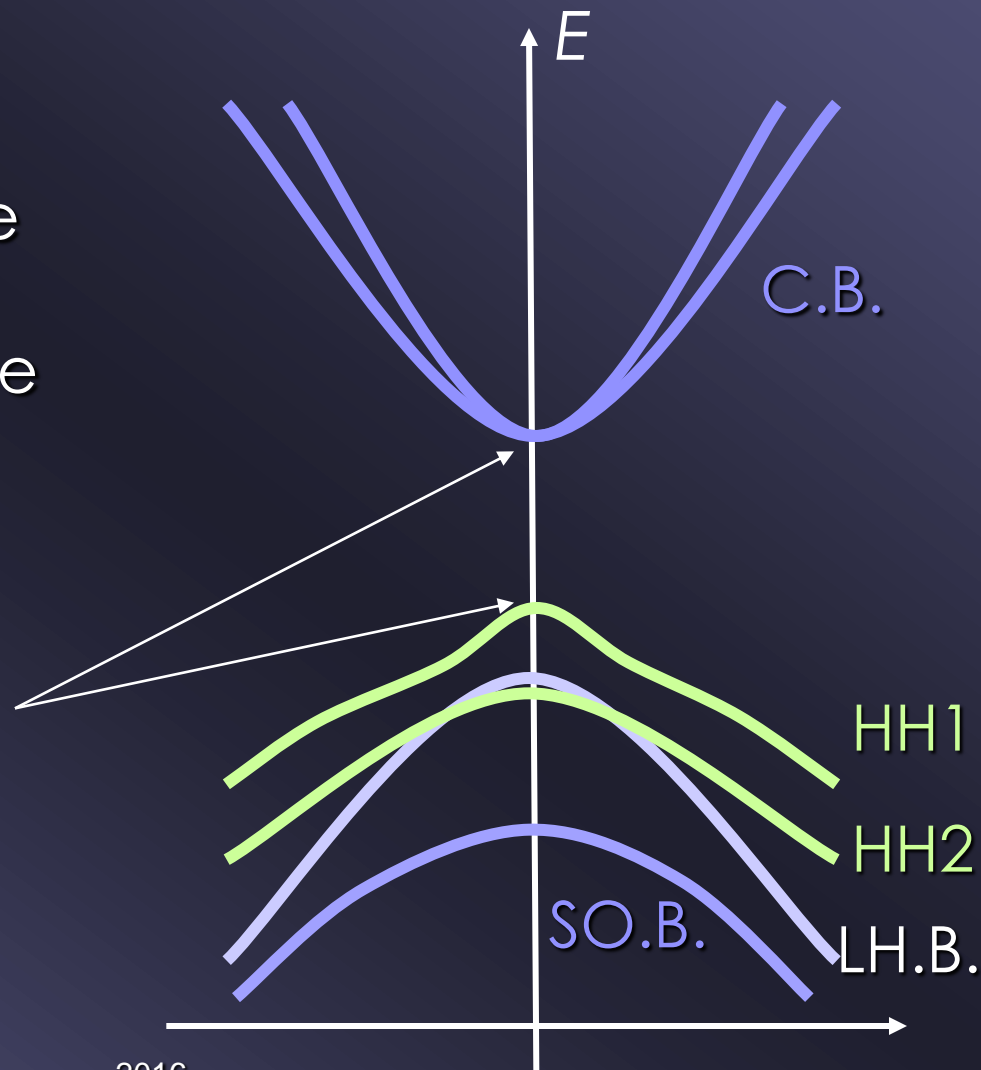


In the Auger recombination the energy which is released via an electron-hole recombination is absorbed by an other hole, which dissipates the energy by generating lattice oscillations (phonons)



Quantum wells cause splitting in the conduction band, lift the degeneracy of the heavy hole and light hole bands, and distort the shape

→ Similar effective mass (curvature) means more effective population inversion (smaller threshold)

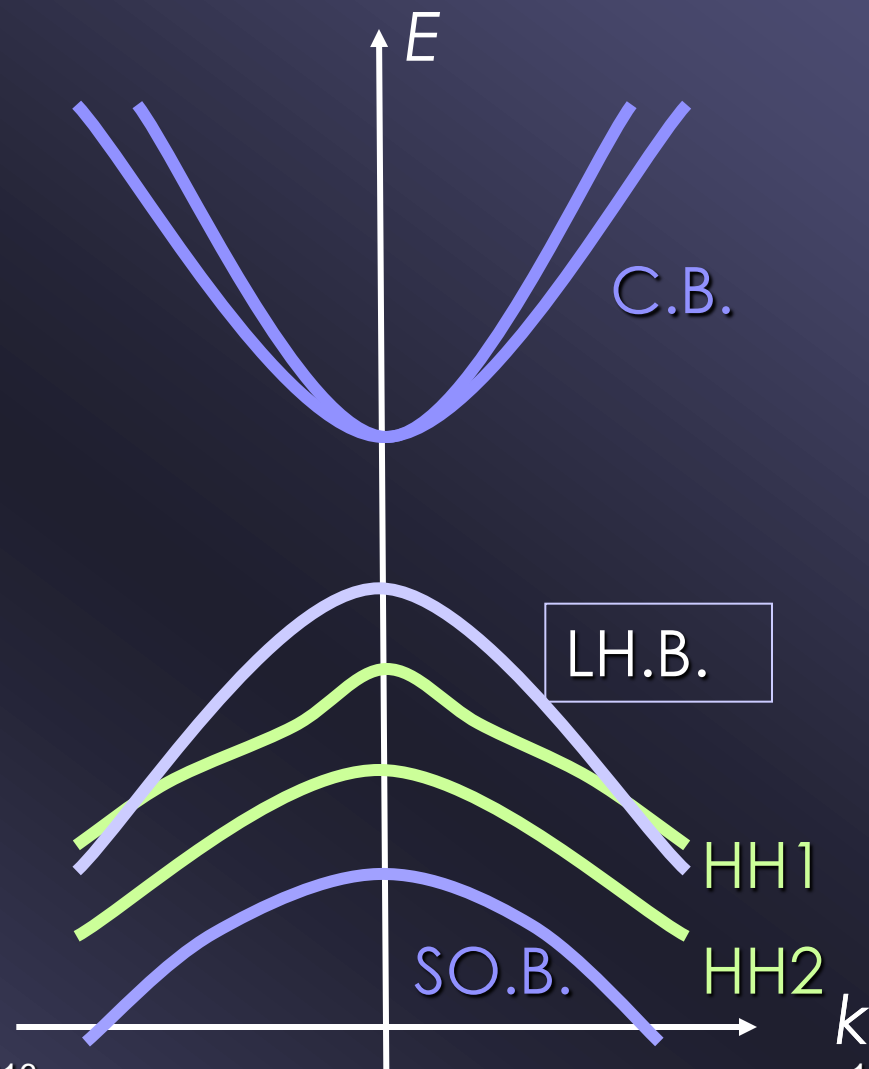


Quantum wells + tensile strain lifts the light hole bands

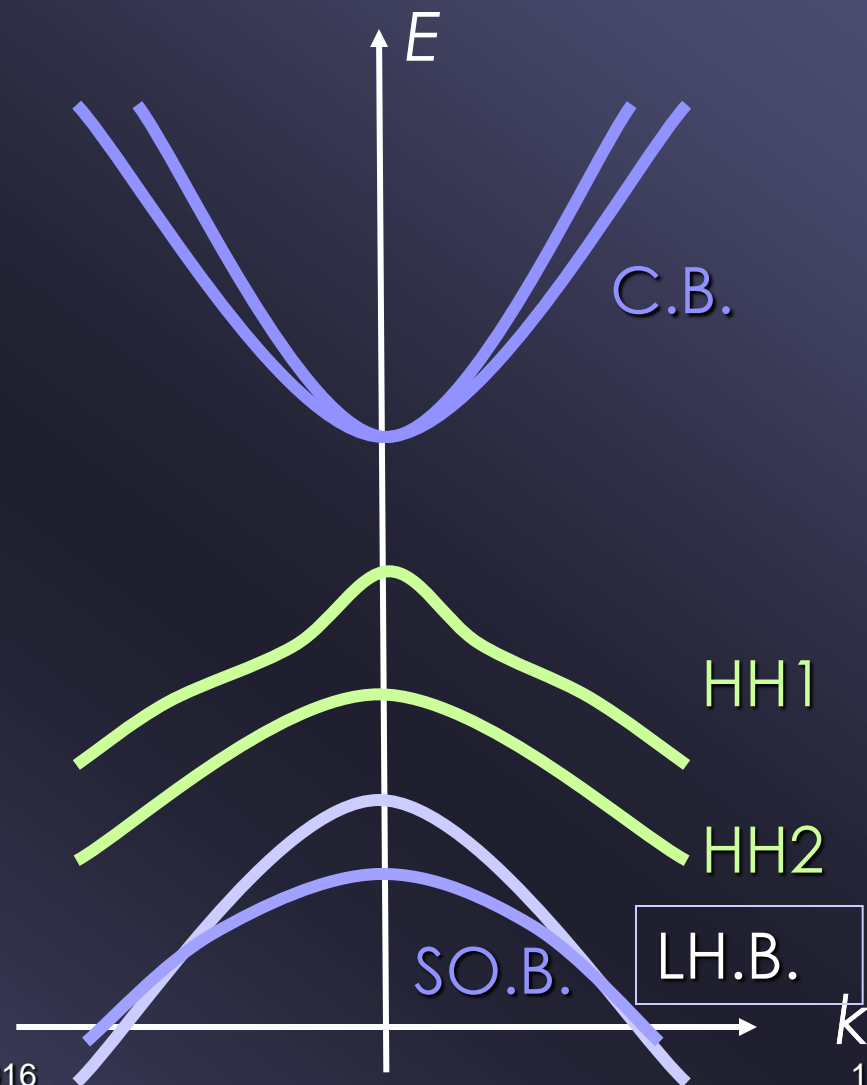
→ TM mode

The split off band is also depressed

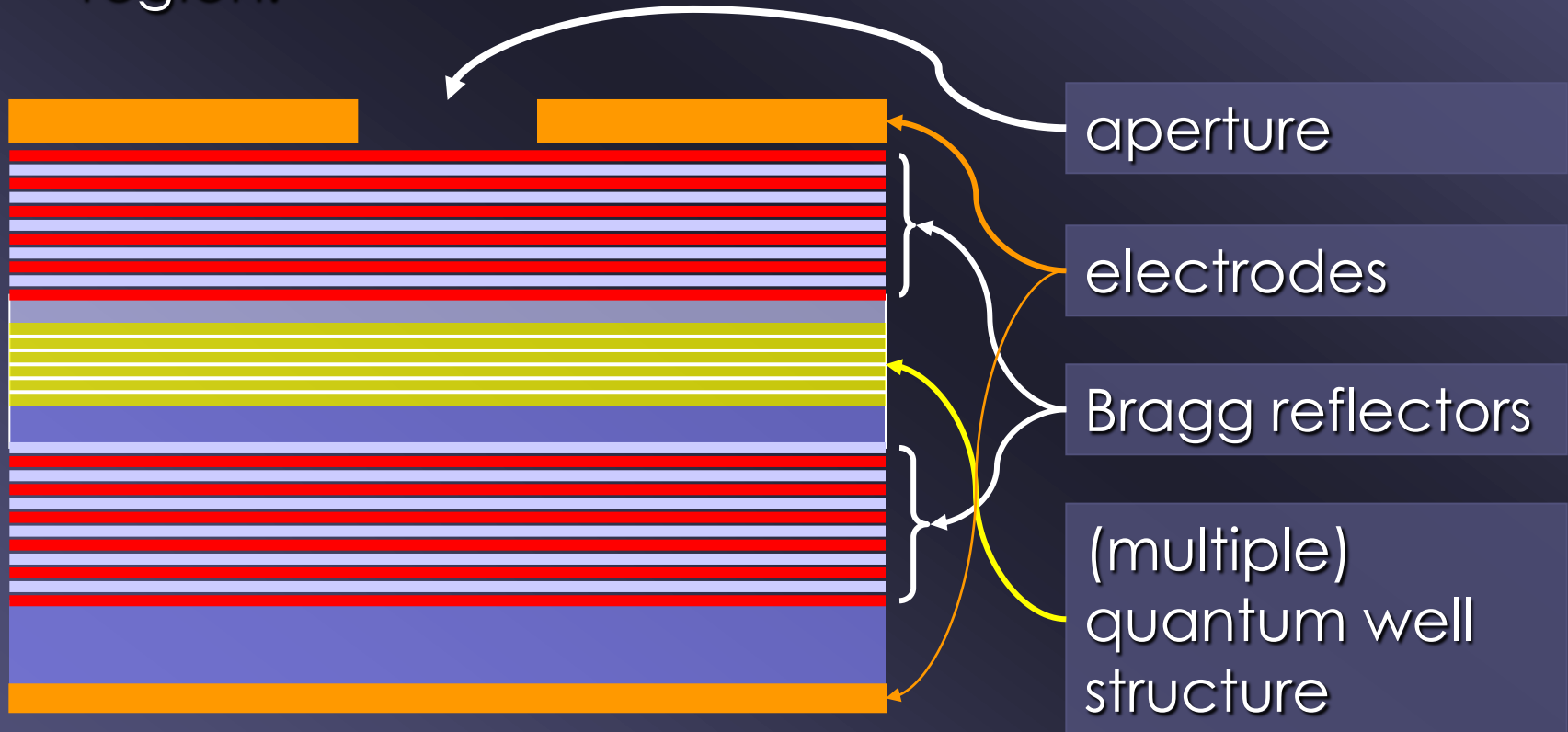
→ less Auger recombination, higher carrier density is possible



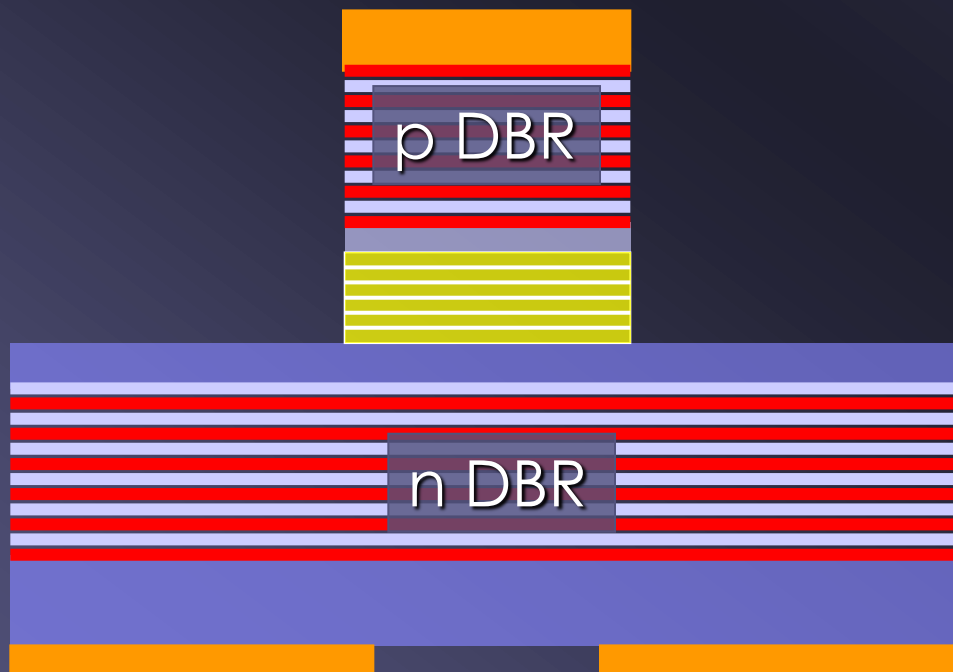
Quantum wells +
compressive strain
 depresses the light hole
 bands, and further
 reduce the heavy hole
 band's curvature
 → TE modulation and
 further decrease in
 threshold level



The high gain of quantum wells make possible to place the resonator above and under the active region:



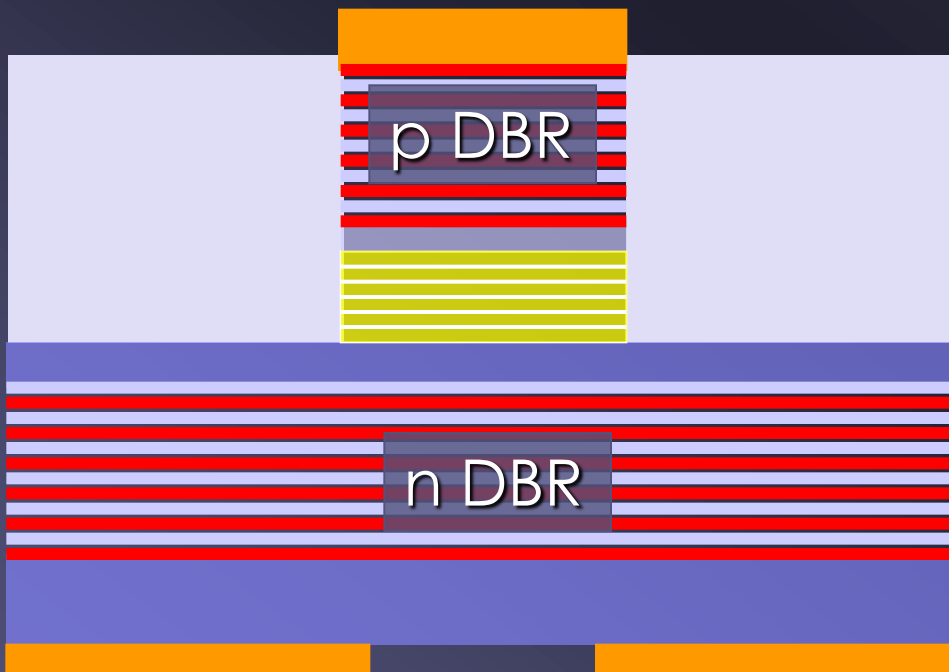
The confinement of population inversion in the y and z dimensions is necessary



etched mesa/air post
VCSEL

aperture usually at
the bottom

The confinement of population inversion in the y and z dimensions is necessary



the etched regions are regrown epitaxially

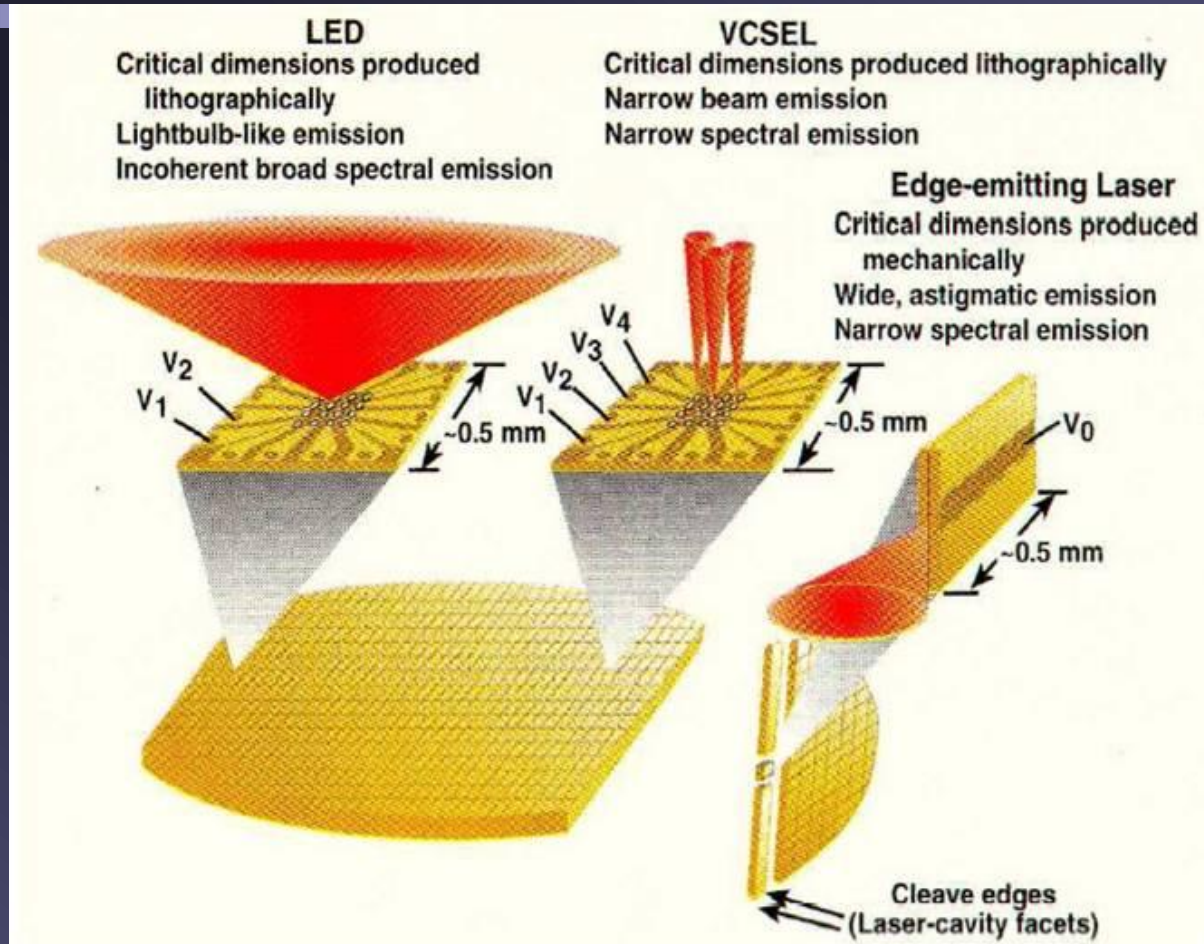
(e.g., high index nipi layers – passive antiguide region)

buried regrowth
VCSELS

Since the reflectors are grown upon the diode structure

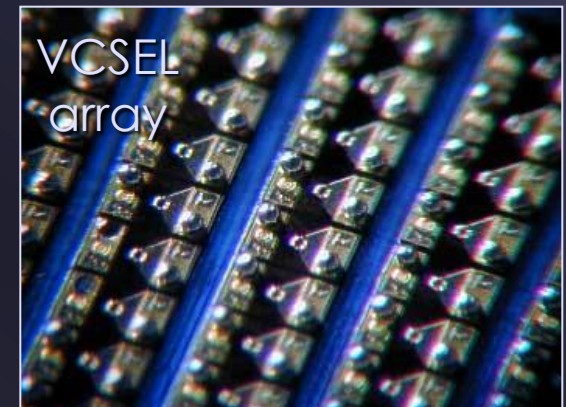
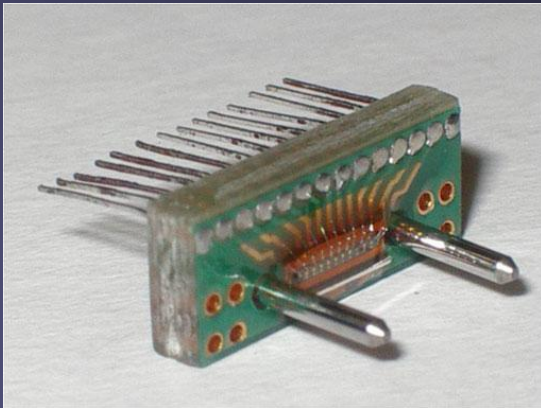
- the resonator length is much shorter than the edge emitting lasers' cavity (less modes)
- the properties of the reflectors can be monitored during the growth
 - ⇒ very good reflectance can be produced
- it is easier to couple the VCSEL's light into an optical fiber
- laser arrays can be produced

LEDs, EELs, VCSELS

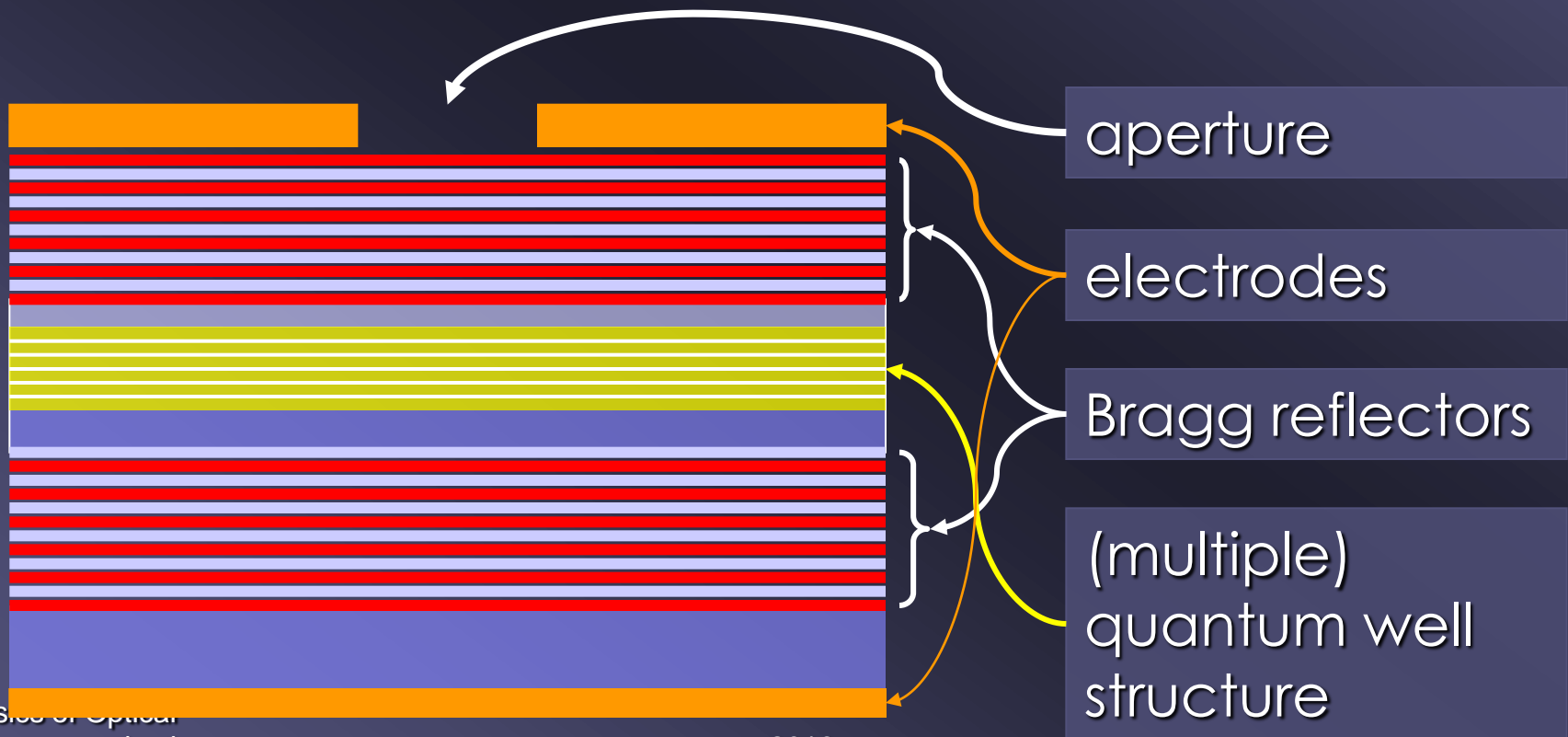


Soda, Iga, Kitahara and Suematsu 1979;
Axel Scherer and Jack Jewell, 1988

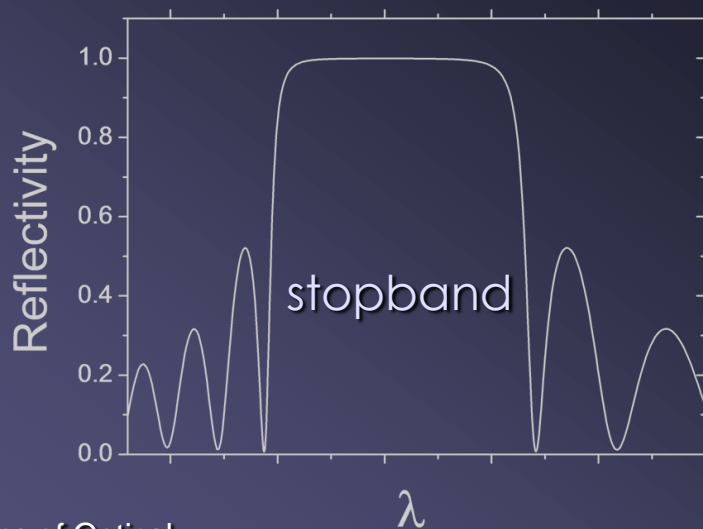
- low electric power consumption
- capability of on-wafer testing
- simplified fiber coupling and packaging
- longitudinal single-mode emission spectrum
- suitability for 2D-array integration, multi-fiber compatibility



The high gain of quantum wells make possible to place the resonator above and under the active region:

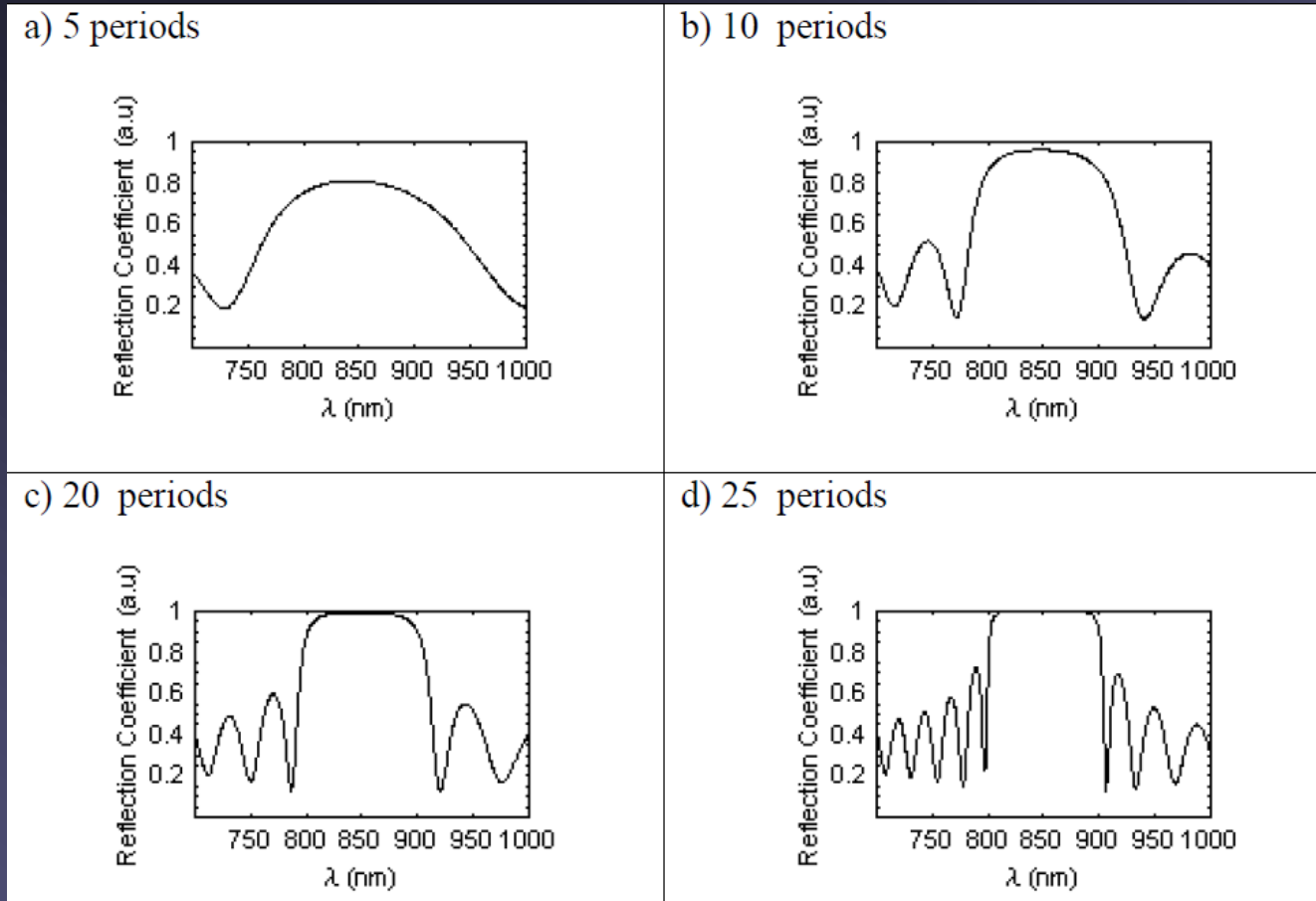


Distributed Bragg reflector is a structure formed from multiple layers of alternating materials with varying refractive index. Each layer boundary causes a partial reflection of an optical wave. For waves whose wavelength is close to $4\times$ the optical thickness of the layers,

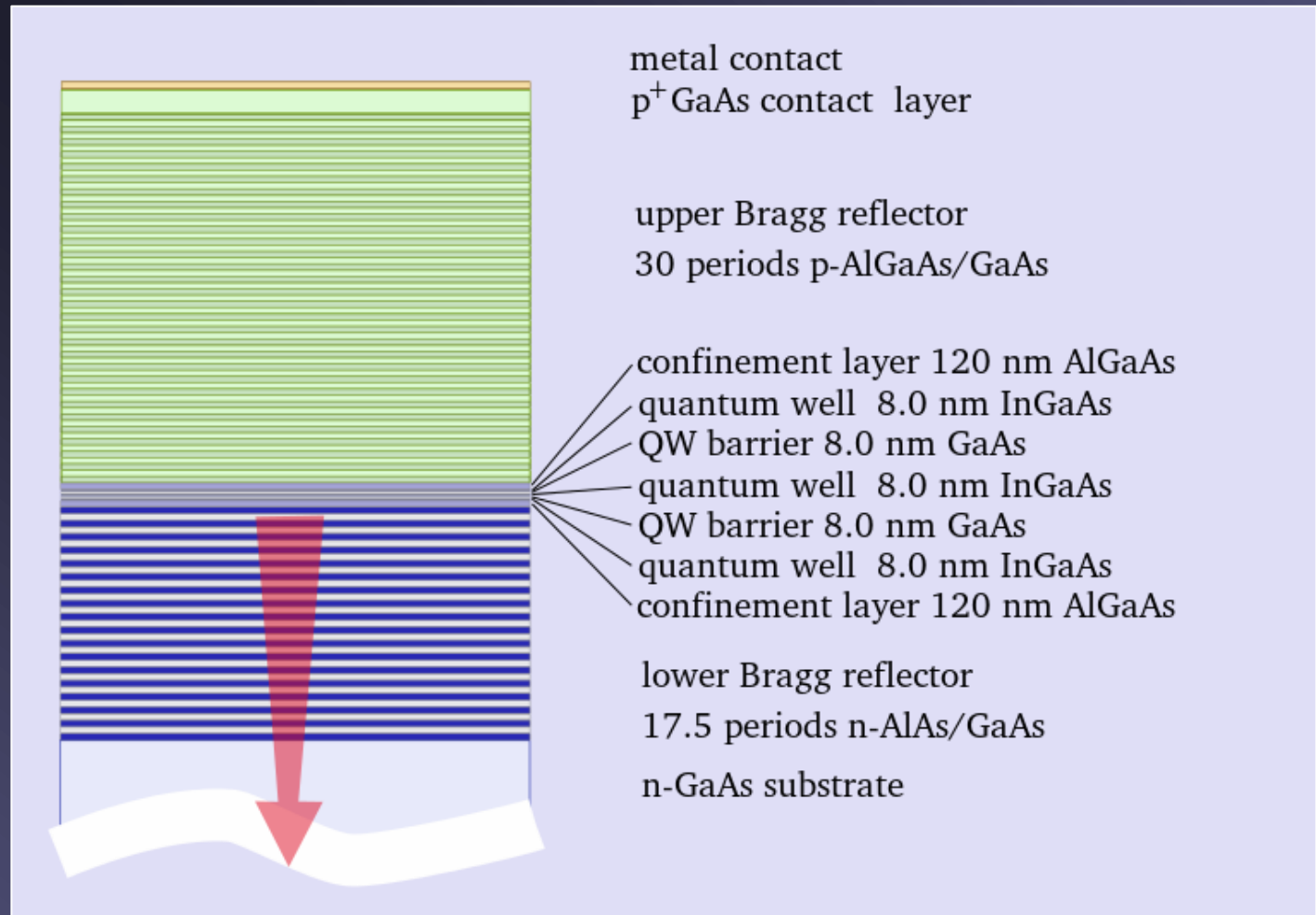


the many reflections combine with constructive interference, and the layers act as a high-quality reflector.

Distributed Bragg reflector



Layers



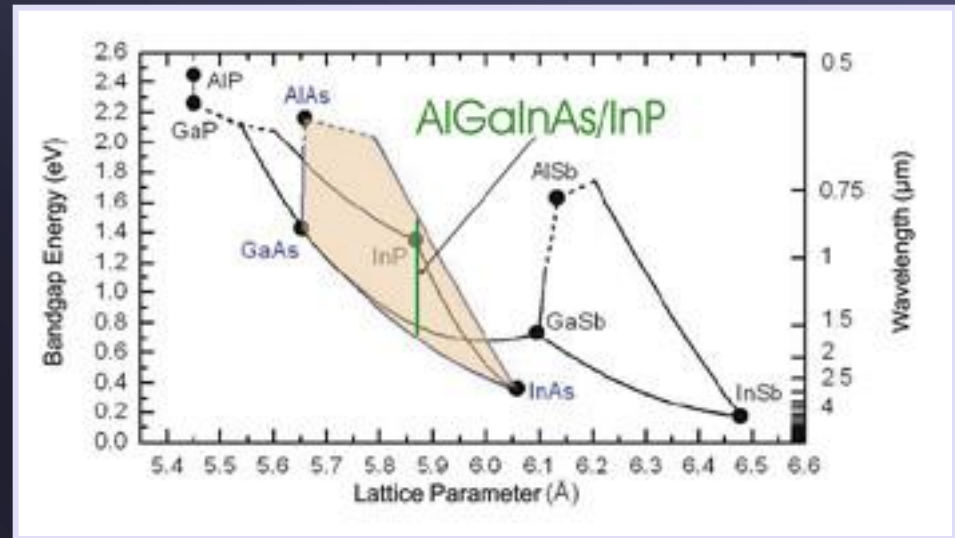
http://en.wikipedia.org/wiki/Vertical-cavity_surface-emitting_laser

Material composition:

- (GaIn)(NAs) on GaAs substrate $1.31\mu\text{m}$,
- (InGaAl)As, (InGa)(AsP), and (AlGa)(AsSb) on InP $1.31\mu\text{m}$ and $1.55\mu\text{m}$

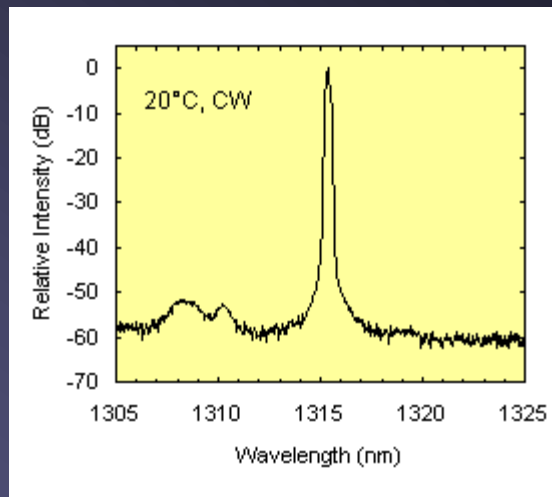
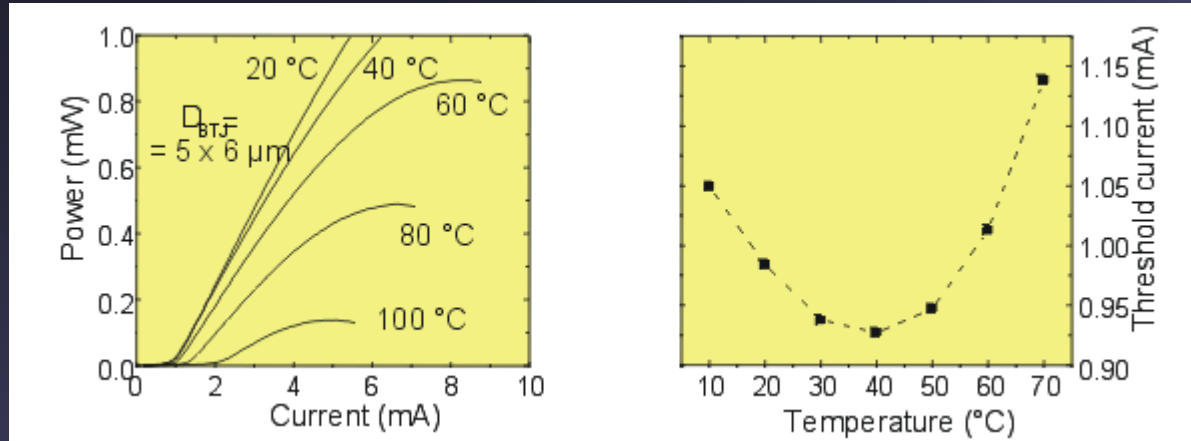
GaAs–AlGaAs system:

- Similar lattice constant
- Strong variation of the refractive index on Al concentration
- Selective oxidation of Al



<http://www.wsi.tum.de/Research/AmanngroupE26/AreasofResearch/SurfaceEmittingLasers/tabid/110/Default.aspx>

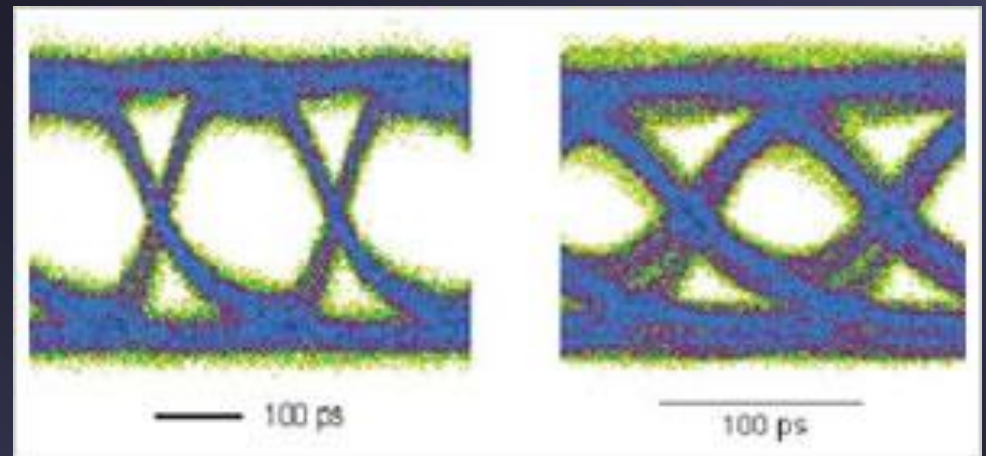
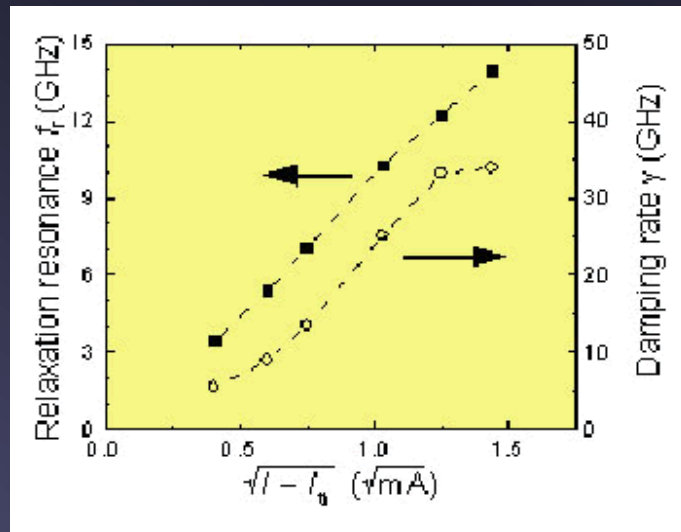
Power characteristics



BTJ-VCSELs at 1.55 μm

<http://www.wsi.tum.de/Research/AmanngroupE26/AreasofResearch/SurfaceEmittingLasers/tabid/110/Default.aspx>

Modulation characteristics



back-to-back measurement at 5 and 10Gbit/s

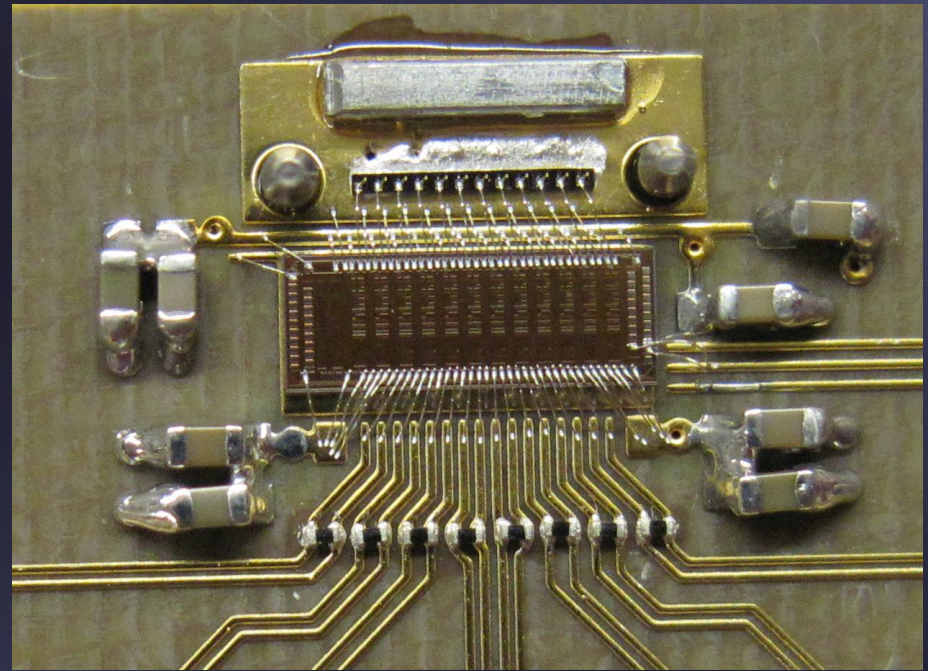
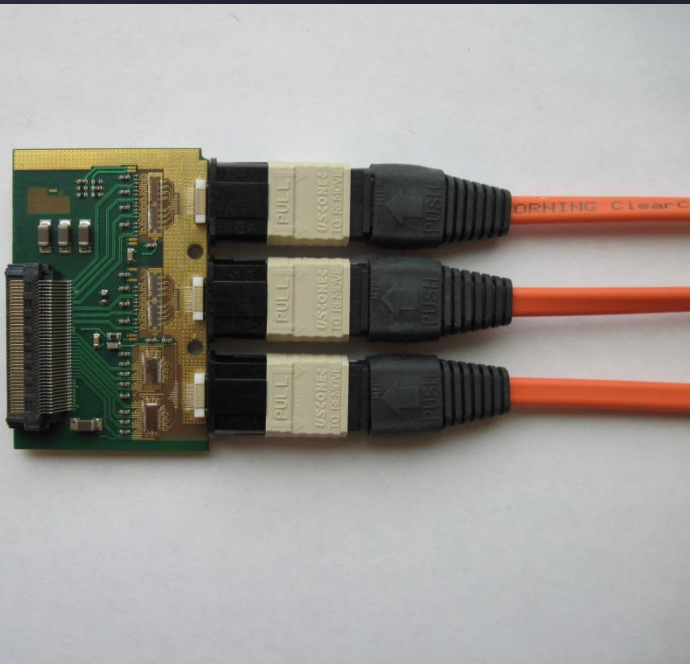
<http://www.wsi.tum.de/Research/AmanngroupE26/AreasofResearch/SurfaceEmittingLasers/tabid/110/Default.aspx>

The multimode MPO and other multifiber systems' sources are usually VCSEL arrays
40 or 100 Gb/s data rate



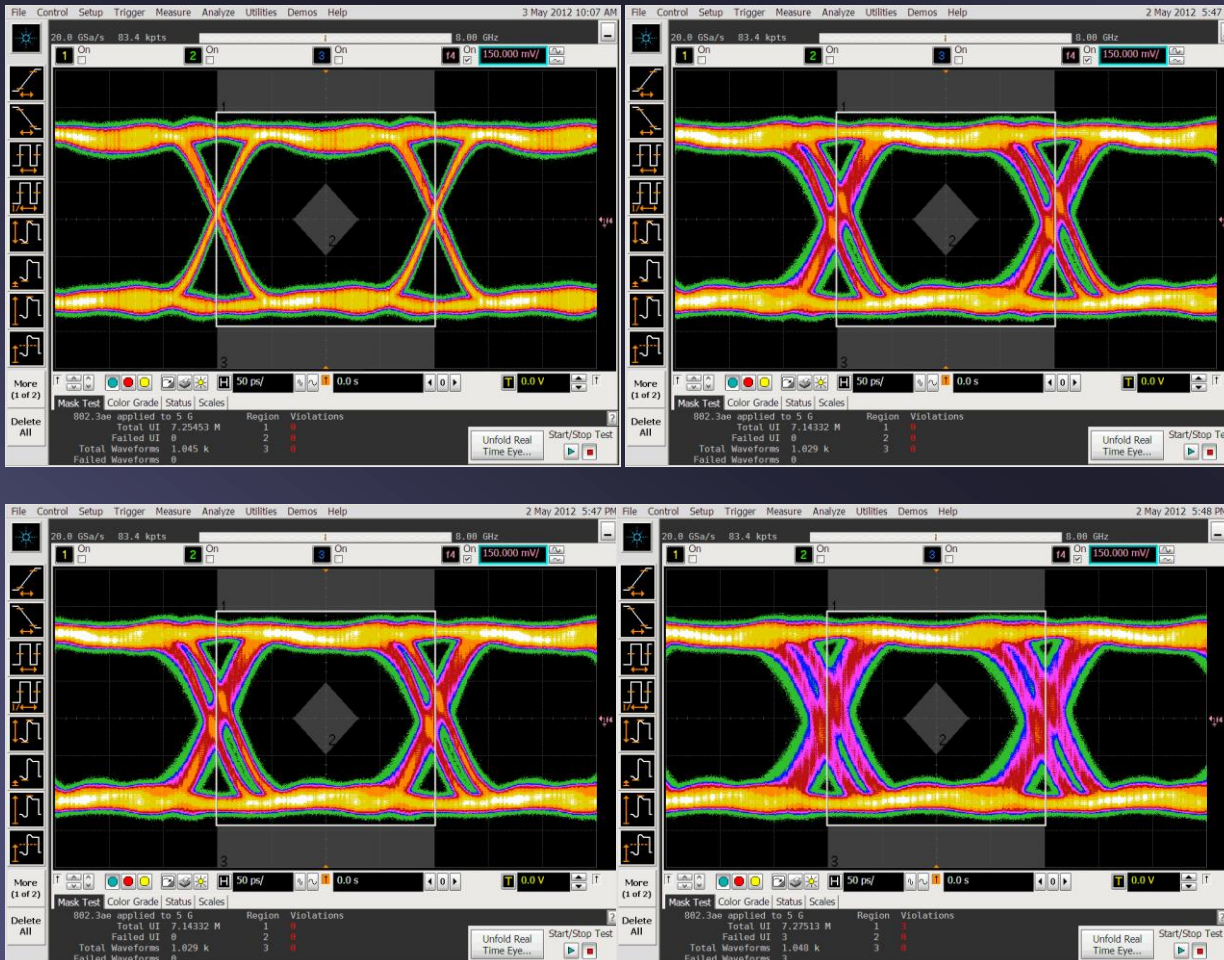
www.elpeus.com,
www.connections.rdm.com

The multimode MPO and other multifiber systems' sources are usually VCSEL arrays



Gan et al. Radiation-Hard/High-Speed VCSEL Array Driver

Eye diagrams



10 Gb/s small form-factor pluggable SFP+ transceiver @ 5 Gb/s with optical loopback / VCSEL driver @ 5 Gb/s after 10 Gb/s SFP+ receiver

One channel active / all channels active

Gan et al. Radiation-Hard/High-Speed VCSEL Array Driver

The confinement of population inversion in the y and z dimensions is necessary, it can be achieved by

- Ion implantation
- Selective oxidation
- Etched mesa – with or without regrowth
- Buried Tunnel Junction

- Amplifiers
 - Erbium doped fibers
 - Raman amplifiers
 - Semiconductor optical amplifiers
- Dispersion compensation
 - Dispersion shifted fibers
 - Dispersion compensating fiber
 - Compact dispersion compensation
- Detectors
 - PIN
 - APD

The 4f (5f) orbitals of the **rare earth metals** are special:

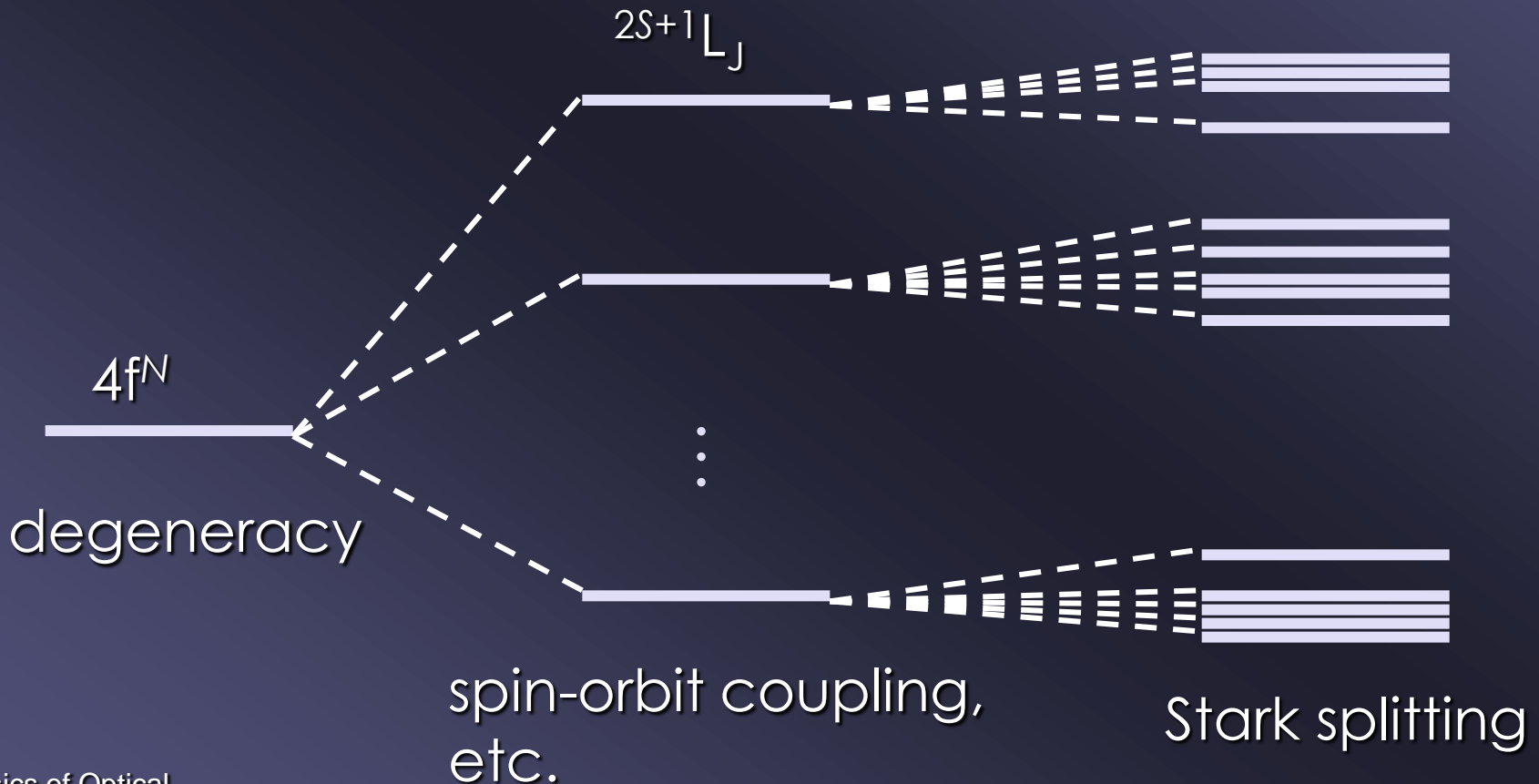
- the electronic structure is $[Xe]4f^{N-1}5d^16s^2$ or $[Xe]4f^N6s^2$

| | | | | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|--|--|--|----|--|
| 1. | H | | | | | | | | | | | | | | | | | | | | He | |
| 2. | Li | Be | | | | | | | | B | C | N | O | F | | | | | | | Ne | |
| 3. | Na | Mg | | | | | | | | Al | Si | P | S | Cl | | | | | | | Ar | |
| 4. | K | Ca | Sc | | | | | | | | | | | | | | | | | | | |
| | | | | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga | Ge | As | Se | Br | | | | Kr | |
| 5. | Rb | Sr | Y | | | | | | | | | | | | | | | | | | | |
| | | | | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In | Sn | Sb | Te | I | | | | Xe | |
| 6. | Cs | Ba | La | | | | | | | | | | | | | | | | | | | |
| | | | | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg | Tl | Pb | Bi | Po | At | | | | Rn | |
| 7. | Fr | Ra | Ac | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | |
| | | | | Ce | Pr | Nd | Pm | Sm | Eu | Gd | Tb | Dy | Ho | Er | Tm | Yb | Lu | | | | | |
| | | | | Th | Pa | U | Np | Pu | Am | Cm | Bk | Cf | Es | Fm | Md | No | Lr | | | | | |

The 4f (5f) orbitals of the **rare earth metals** are special:

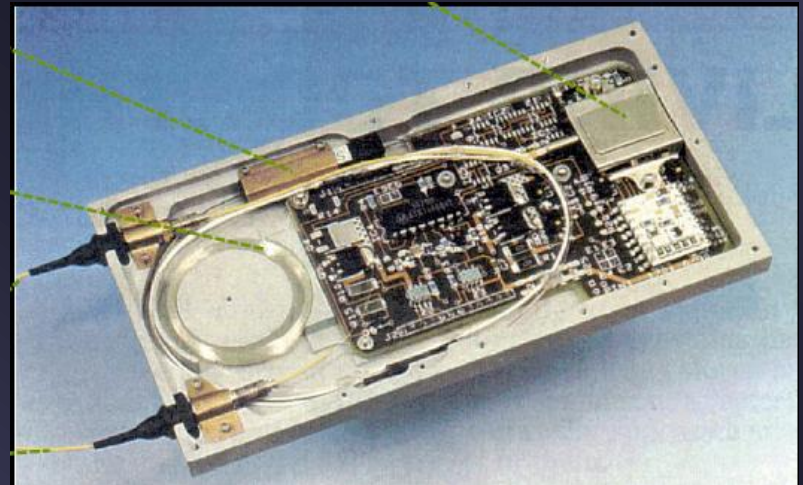
- the electronic structure is $[\text{Xe}]4f^{N-1}5d^16s^2$ or $[\text{Xe}]4f^N6s^2$
- they are usually 3+ ions
- $5s^25p^6$ orbitals have larger radius, than the 4f
 \longrightarrow isolating “sphere” \longrightarrow atom-like behavior
- energy spectrum of very narrow bands if the insulator is doped by lanthanoids

The 4f orbitals of the **rare earth metals** is split by atomic forces and the crystalline field

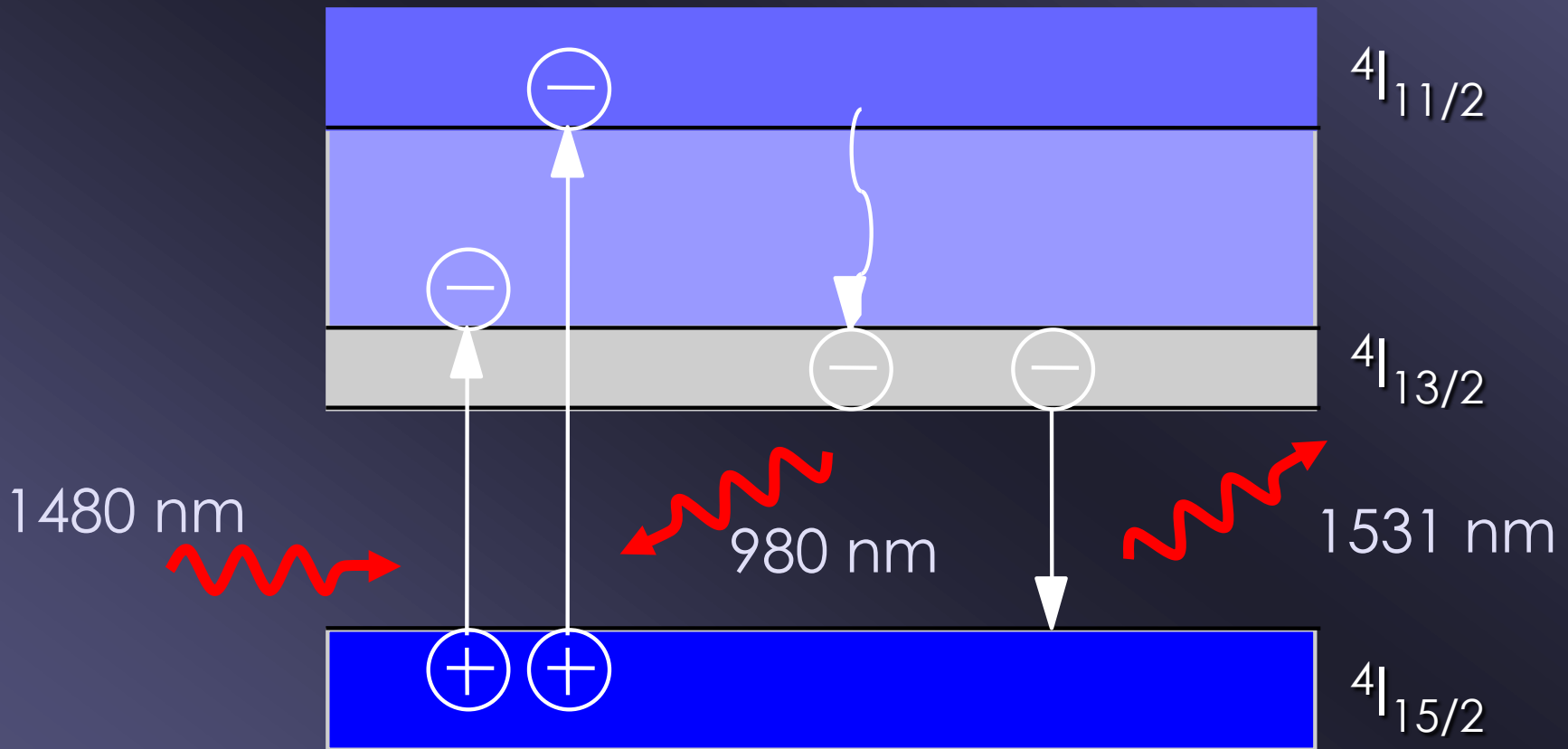


The ${}^4I_{13/2} \leftrightarrow {}^4I_{15/2}$ (GS) transition in Er^{3+} ions belong to photons of wavelength $\sim 1.5 \mu\text{m}$

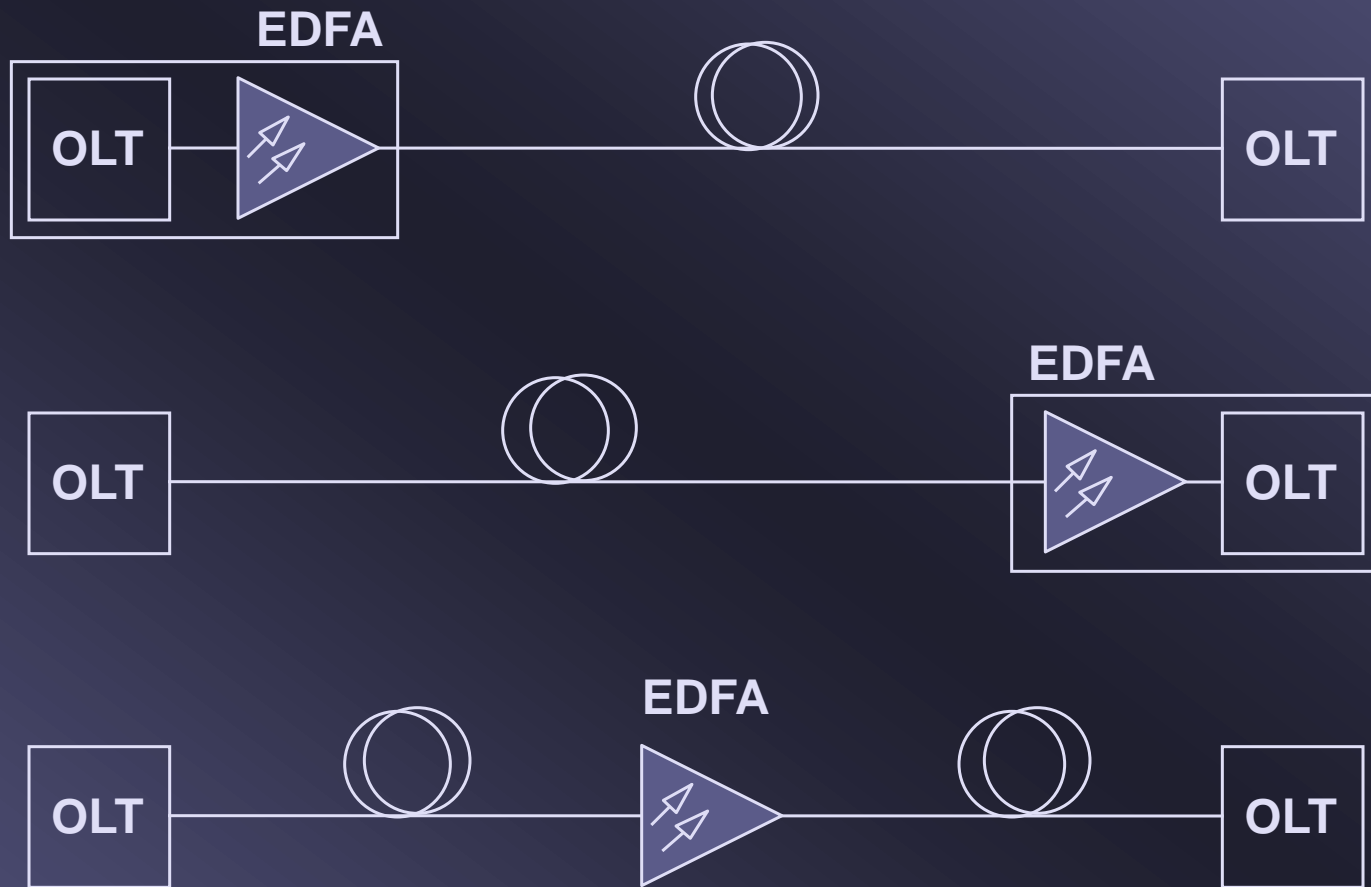
- two main pump regions: 1480 nm and 980 nm with significant absorption
- large gap between the two lowest level ${}^4I_{13/2}$ and ${}^4I_{11/2} \implies$ large lifetime of the ${}^4I_{13/2}$ (~ 10 ms, depending on hosts), mostly radiative transition
- three-level system
- concentration quenching \implies shorter lifetime



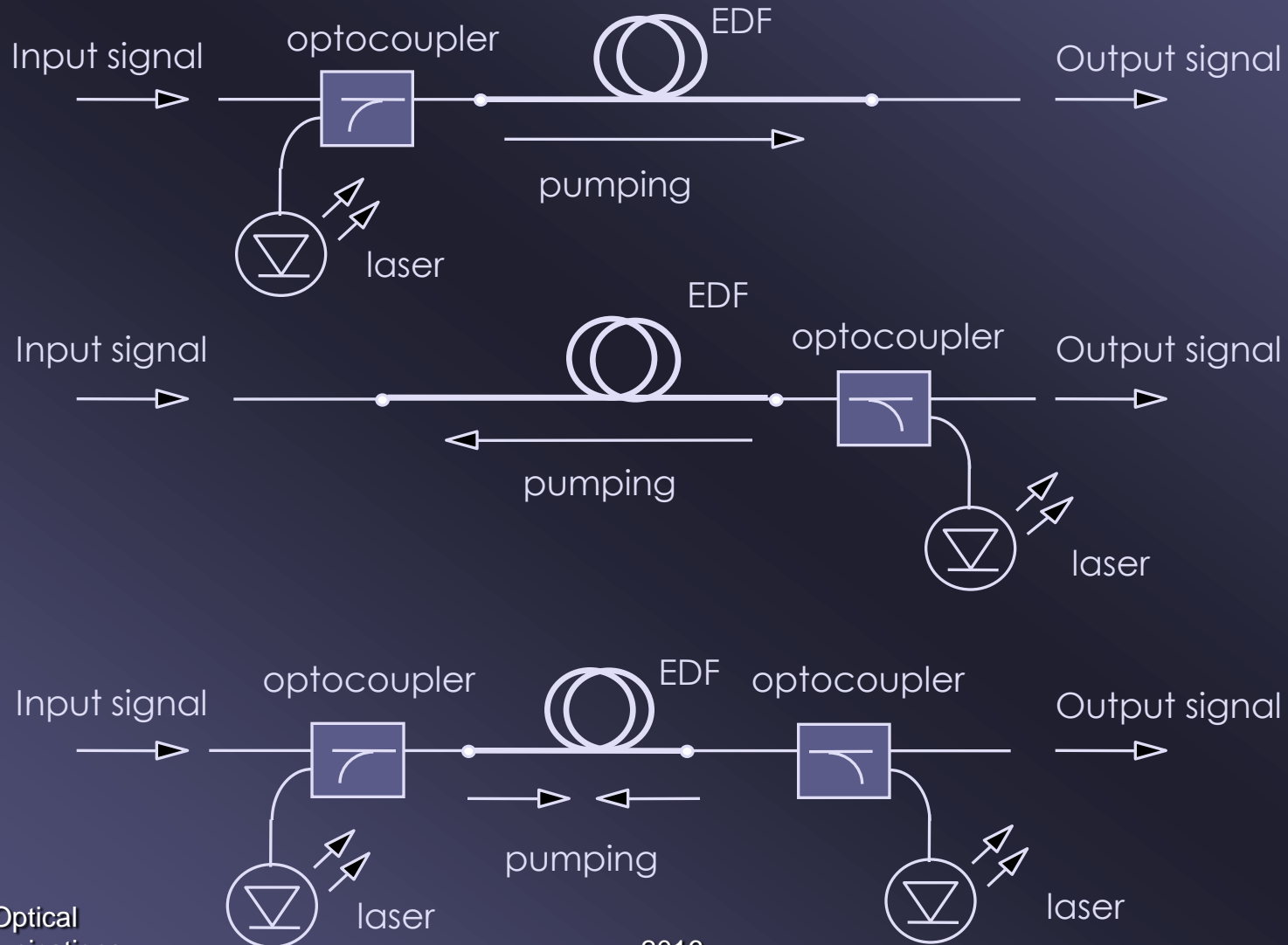
The $4I_{13/2} \leftrightarrow 4I_{15/2}$ (GS) transition in Er^{3+} ions belong to photons of wavelength $\sim 1.5 \mu m$



Application of EDFAs



Pumping of EDFAs

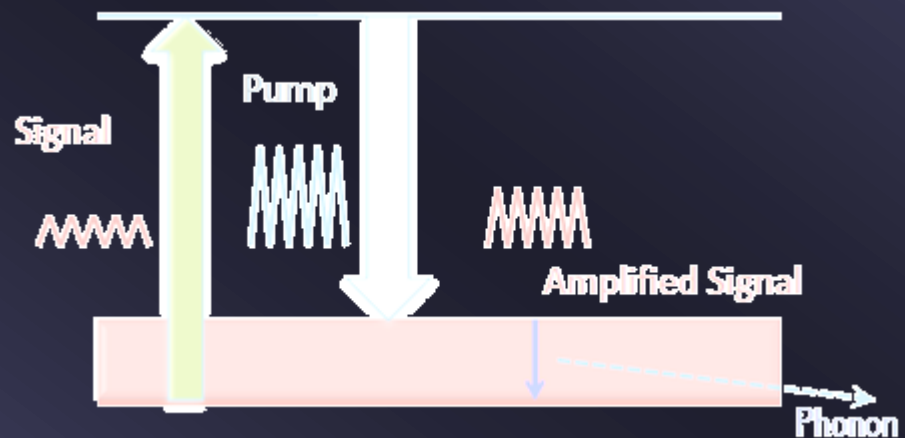


Raman amplifiers

Based on stimulated Raman scattering



- the pumping photon gives part of its energy to the fiber
- energy relaxes as phonon
- better for end amplifier
- backward pumping is usual



Venkataraman: Optical Amplifiers

Based on semiconductor heterojunctions, but not in laser mode

- preventing laser mode by antireflection coating and carefully chosen cleave angle
- electrically pumped
- best for in-line amplifier, compact
- strong nonlinearity
- larger noise
- smaller amplification
- smaller bandwidth



Venkataranaman: Optical Amplifiers

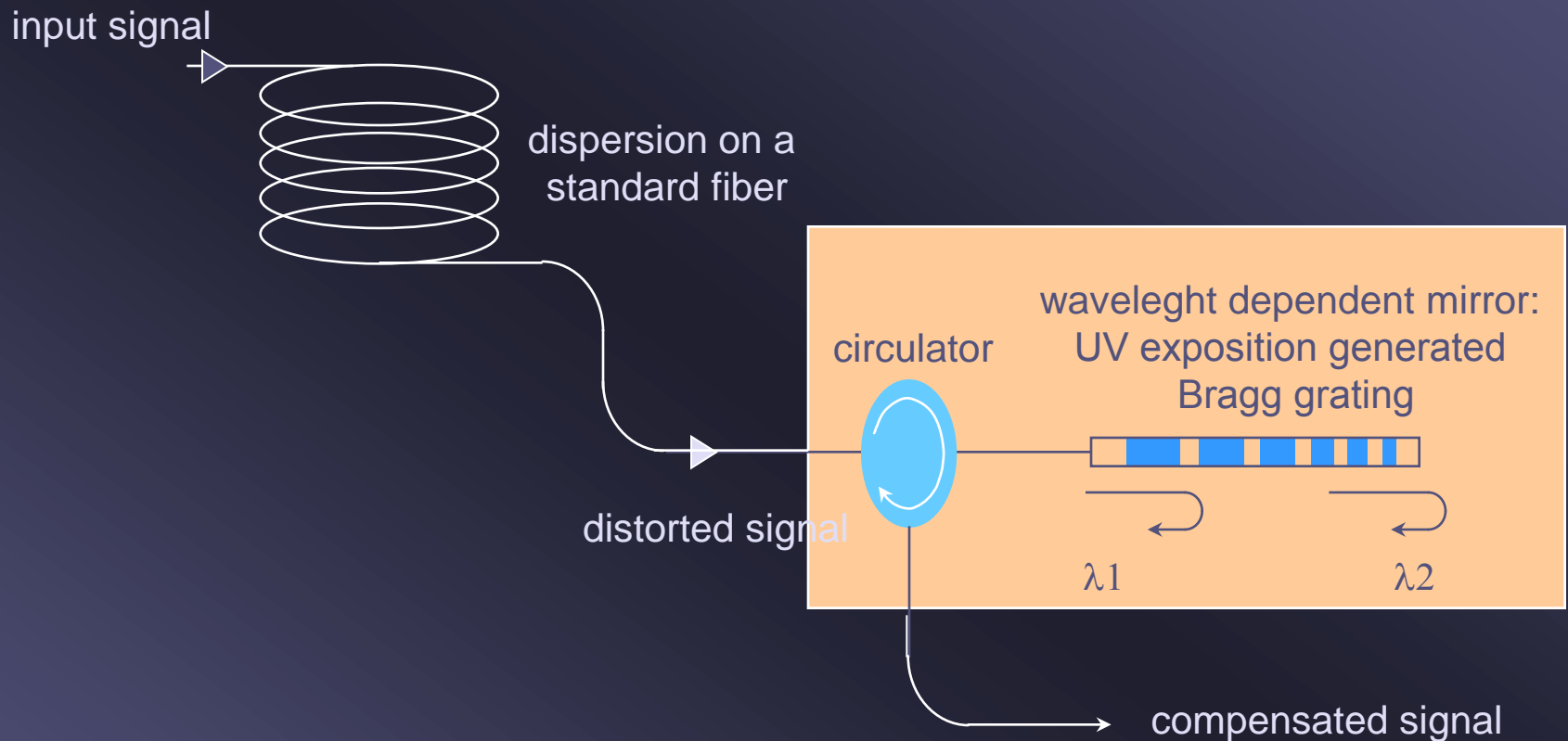
An optical regenerator consists of

- amplifier
- dispersion compensator

Dispersion compensators can be

- dispersion shifted fibers – no need for dispersion compensators
- regular fibers
 - dispersion compensating fibers
 - Bragg grating fiber and circulator

Dispersion compensation

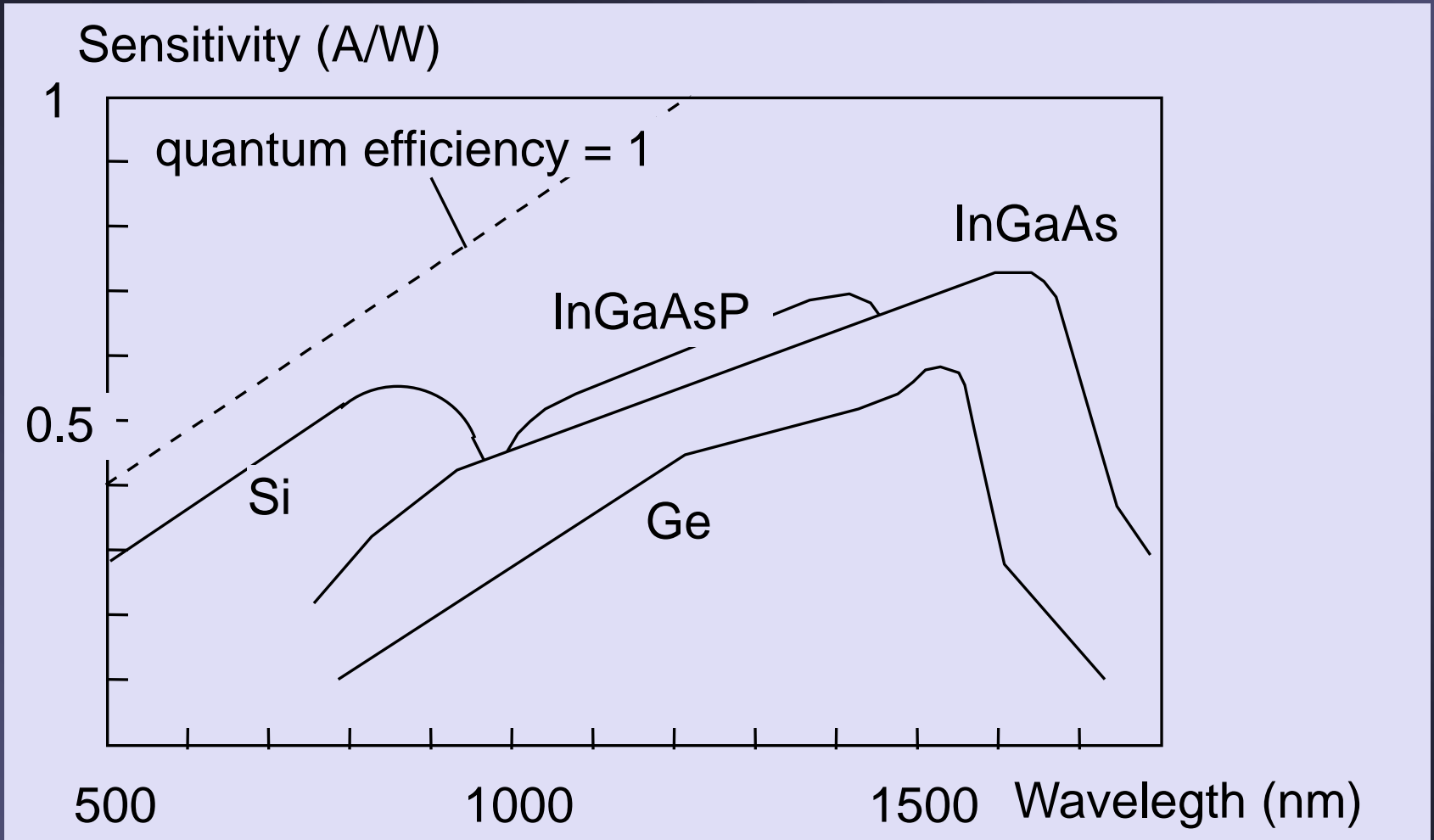


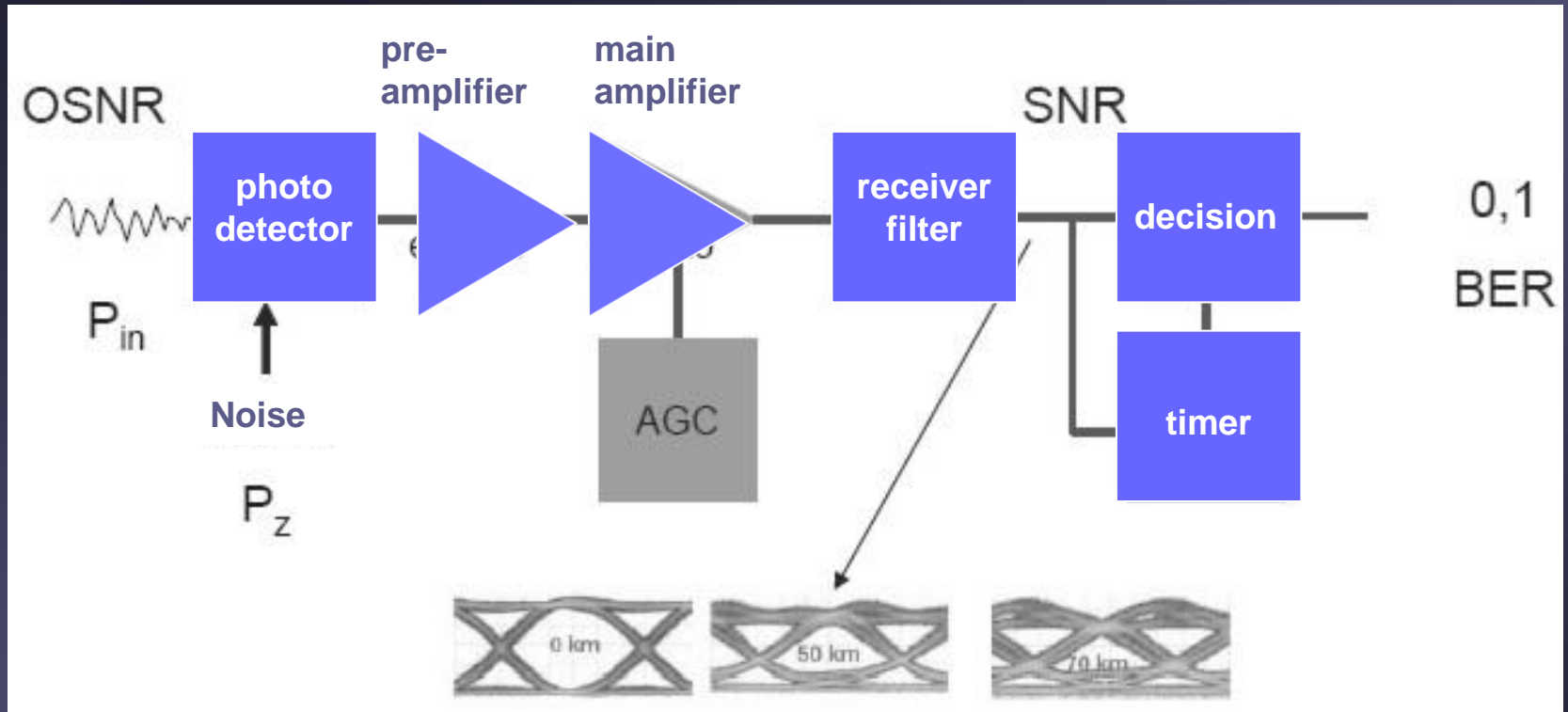
Receivers produce electrons or other charge carriers from photons.

Requirements:

- Large efficiency
 - decreasing reflection – connection
 - increasing detecting area – PIN
 - decreasing the recombination of the generated charge carriers – APD
- Low noise
- Compatibility
- Quick response
- Wavelength selectivity (not necessarily)

- Quantum efficiency: $\eta = J_f / e\Phi$: the rate of the photons and the arising charge carriers
- Sensitivity: $R = e\eta / h\nu$: current arising from the incoming power in the detector
- Bandwidth: depends on the charge carriers' crossing time in the empty
- Noise:
 - dark current
 - shotnoise



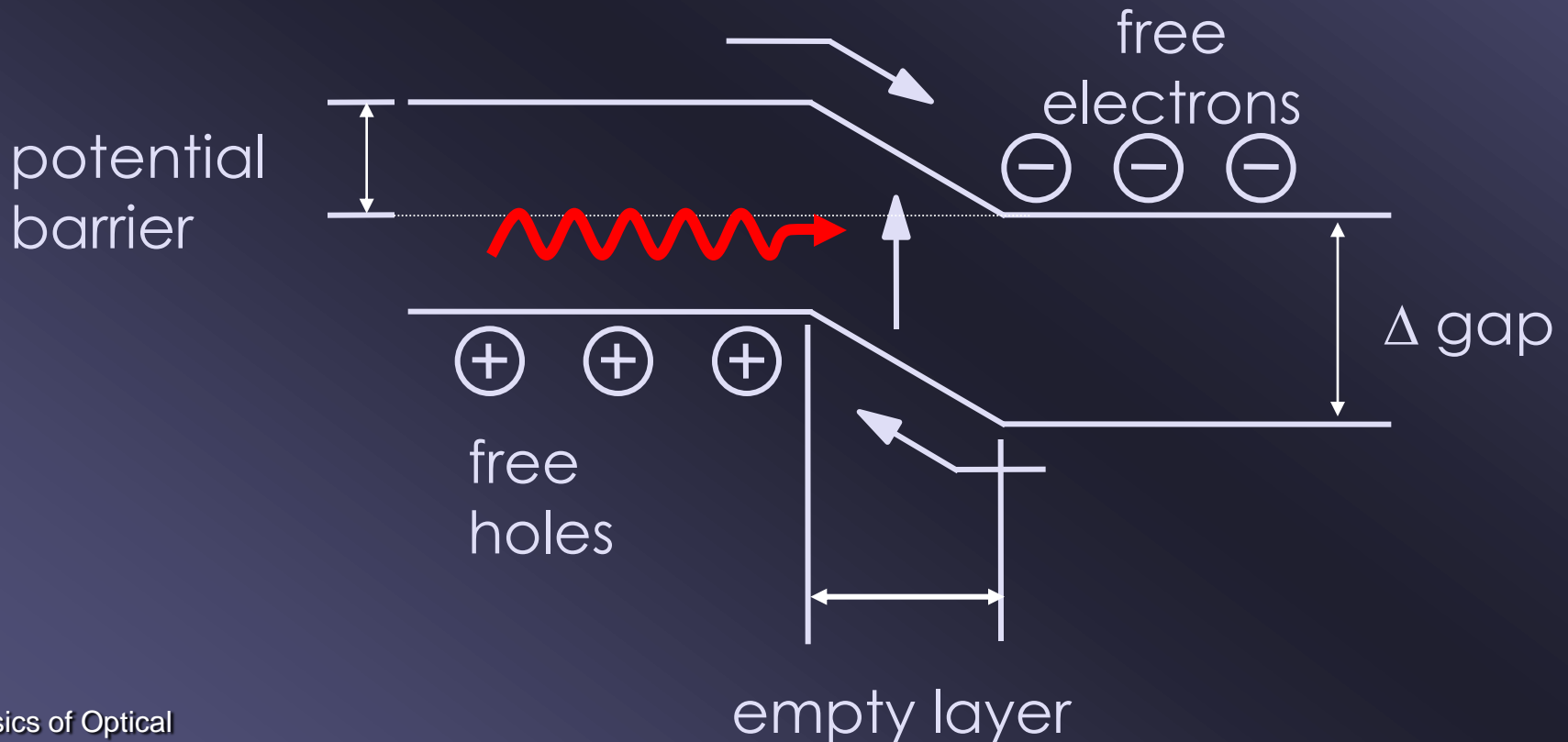


Receiver sensitivity: received optical power necessary for 10^{-9} BER
Quantum limit: 36 photons/bit, practically ~ 1000 photons/bit

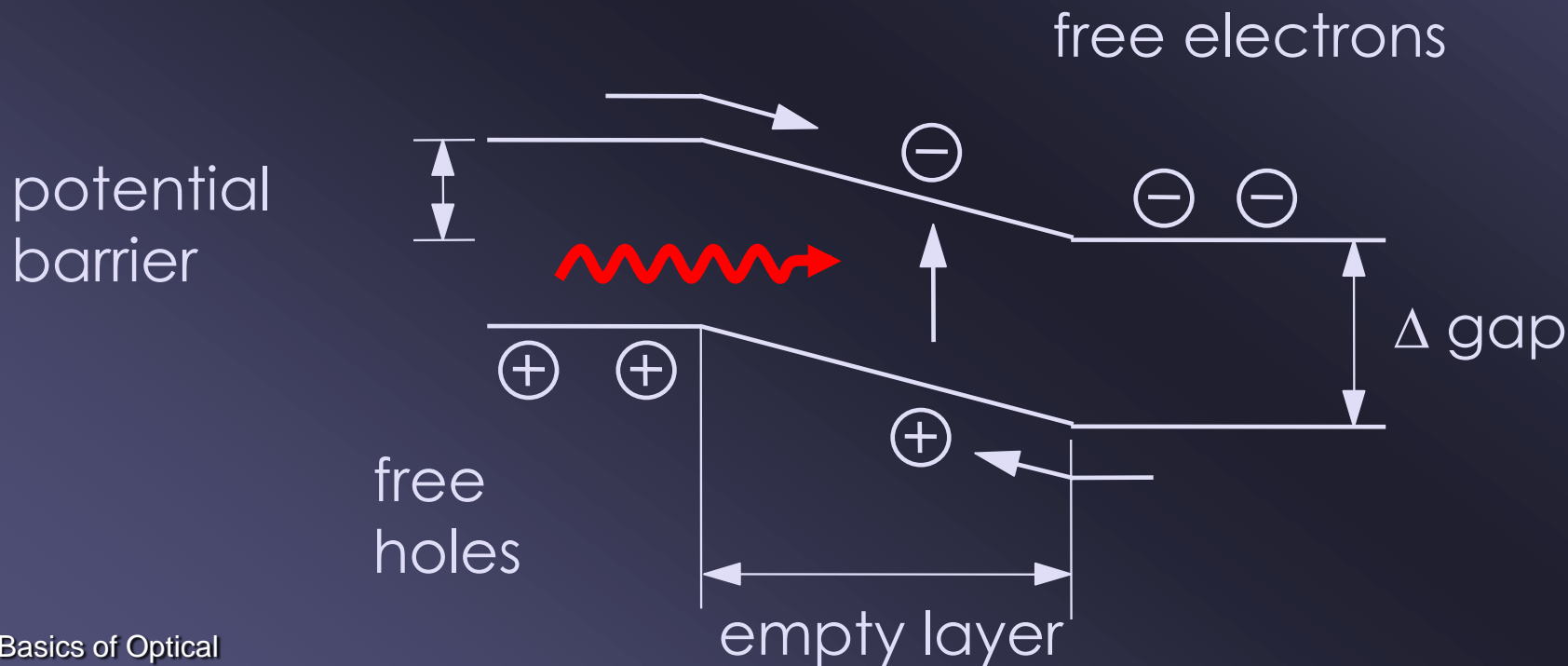
p-n heterojunction photodiode

Reverse bias empty layer between p and n

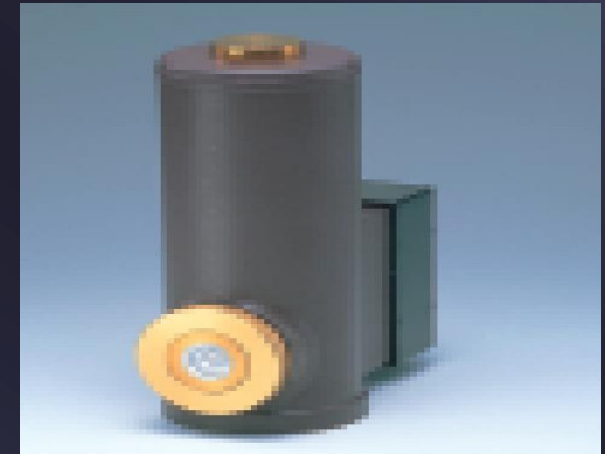
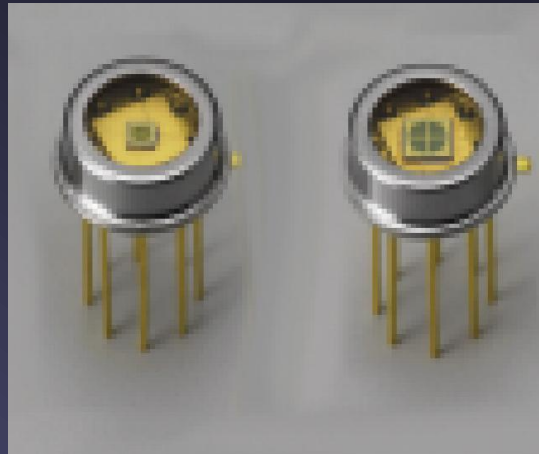
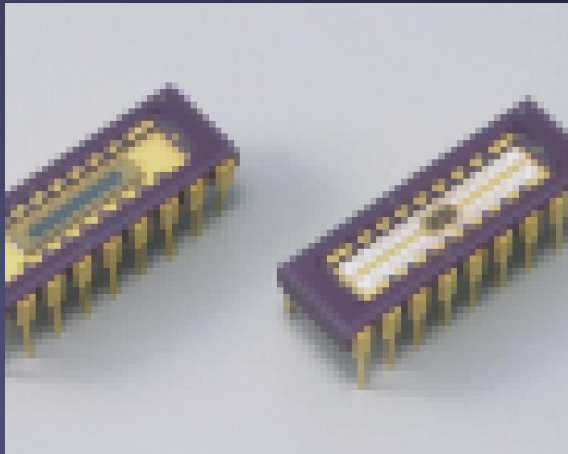
Electron-hole pairs arise due to photon excitations



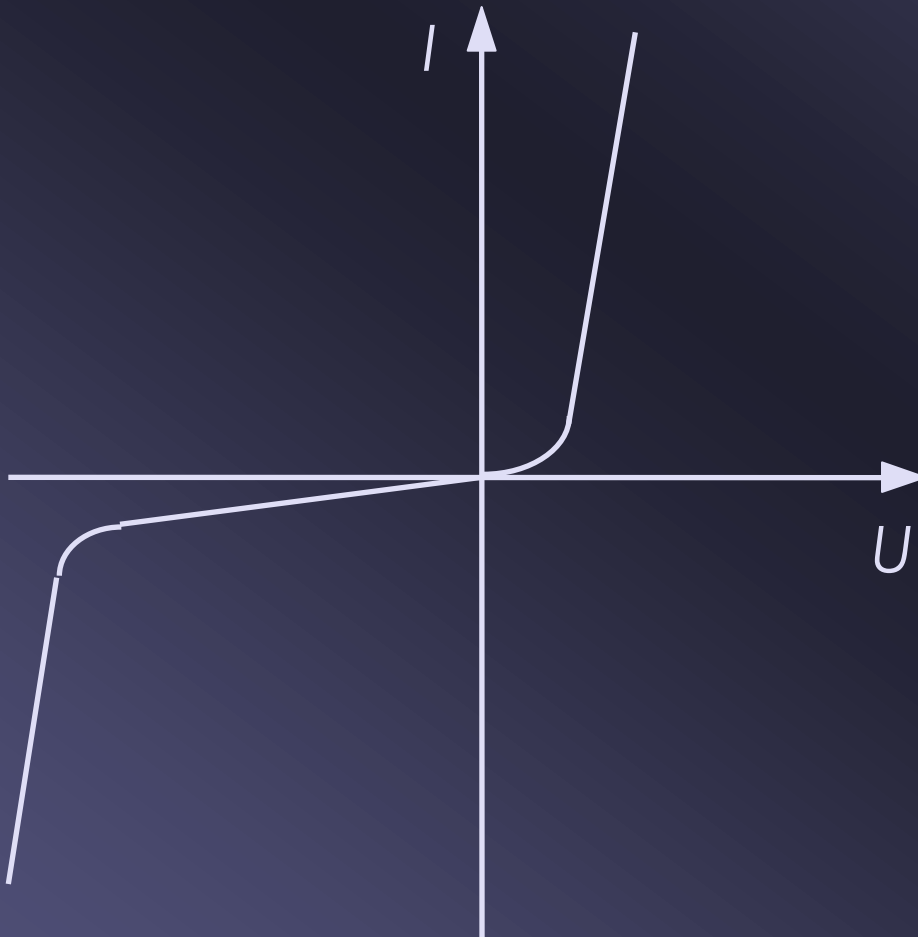
PIN fotodióda



PIN photodiode

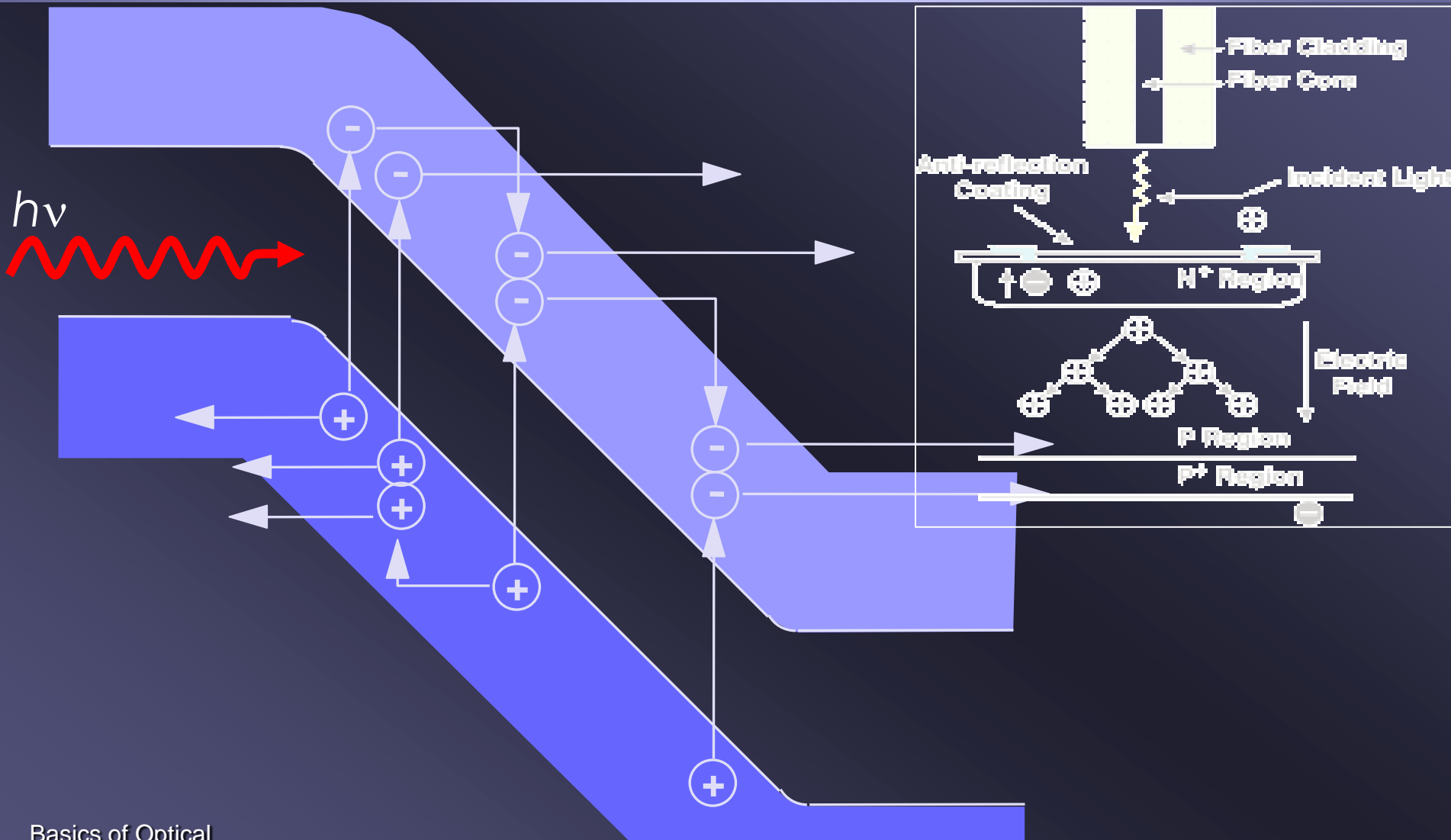


Avalanche photodiodes



Avalanche:
due to the large
voltage, large
electric field
one electron can be
accelerated so that its
kinetic energy can
generate more
electron-hole pairs

Avalanche photodiodes



- Physical basics
 - electrooptic effect
 - magnetooptic effect
 - acoustooptic effect
 - elastooptic effect
 - thermooptic effect
 - Bragg grating, Bragg mirrors
 - interferometers
- Modulation
- Switching

Modulation methods

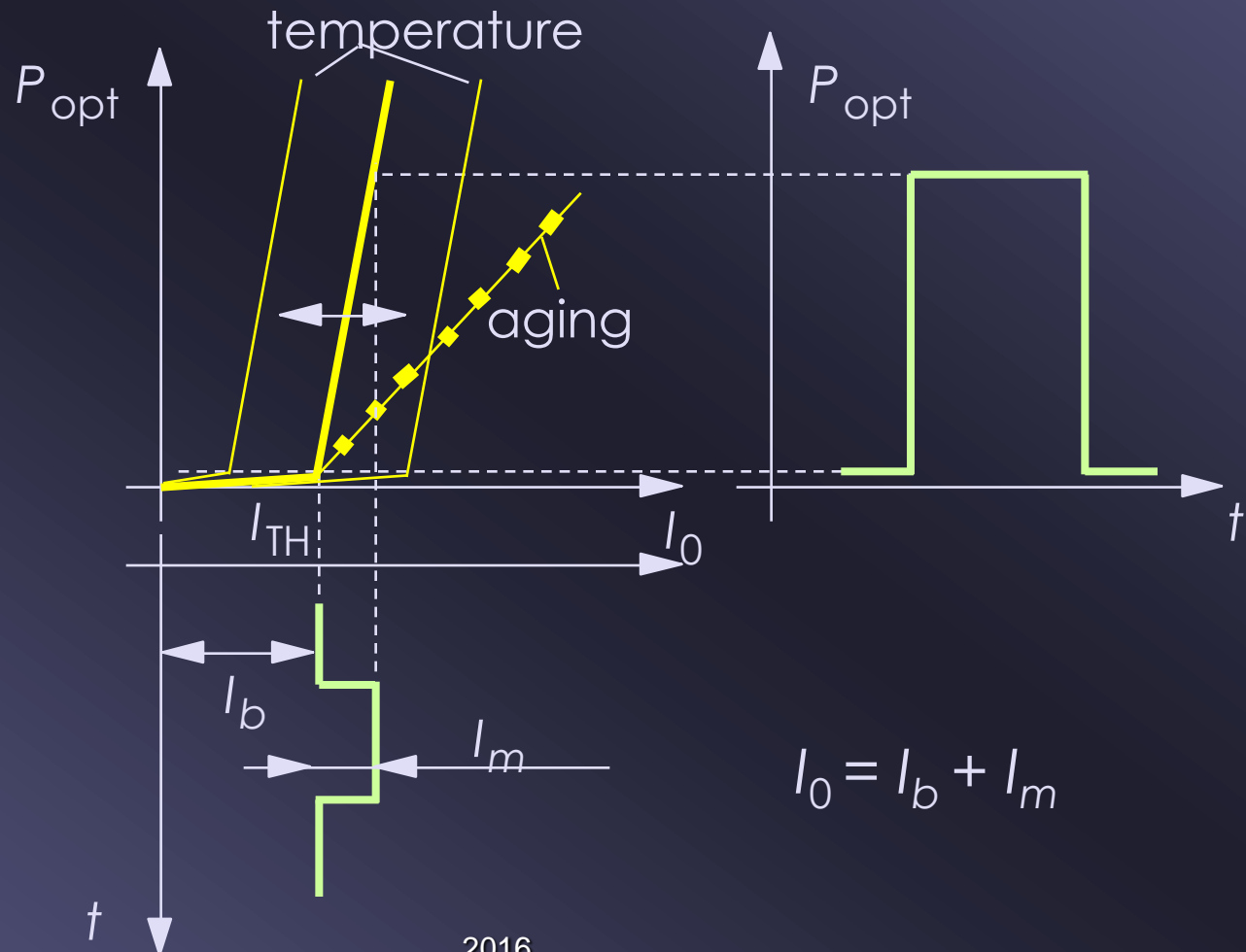
External

- interferometer
- absorption
- reflection

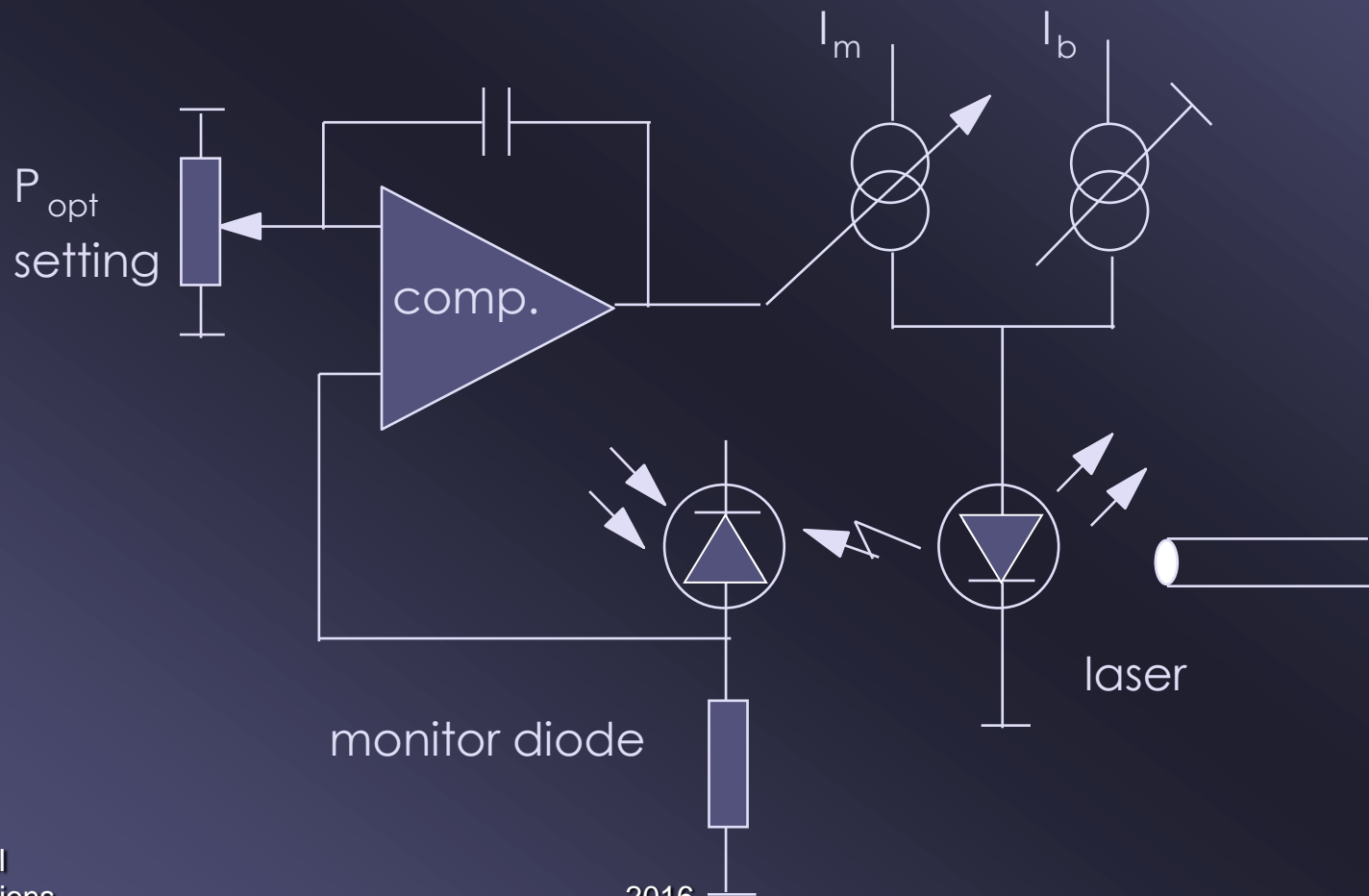
Direct

- internal modulation of the laser current

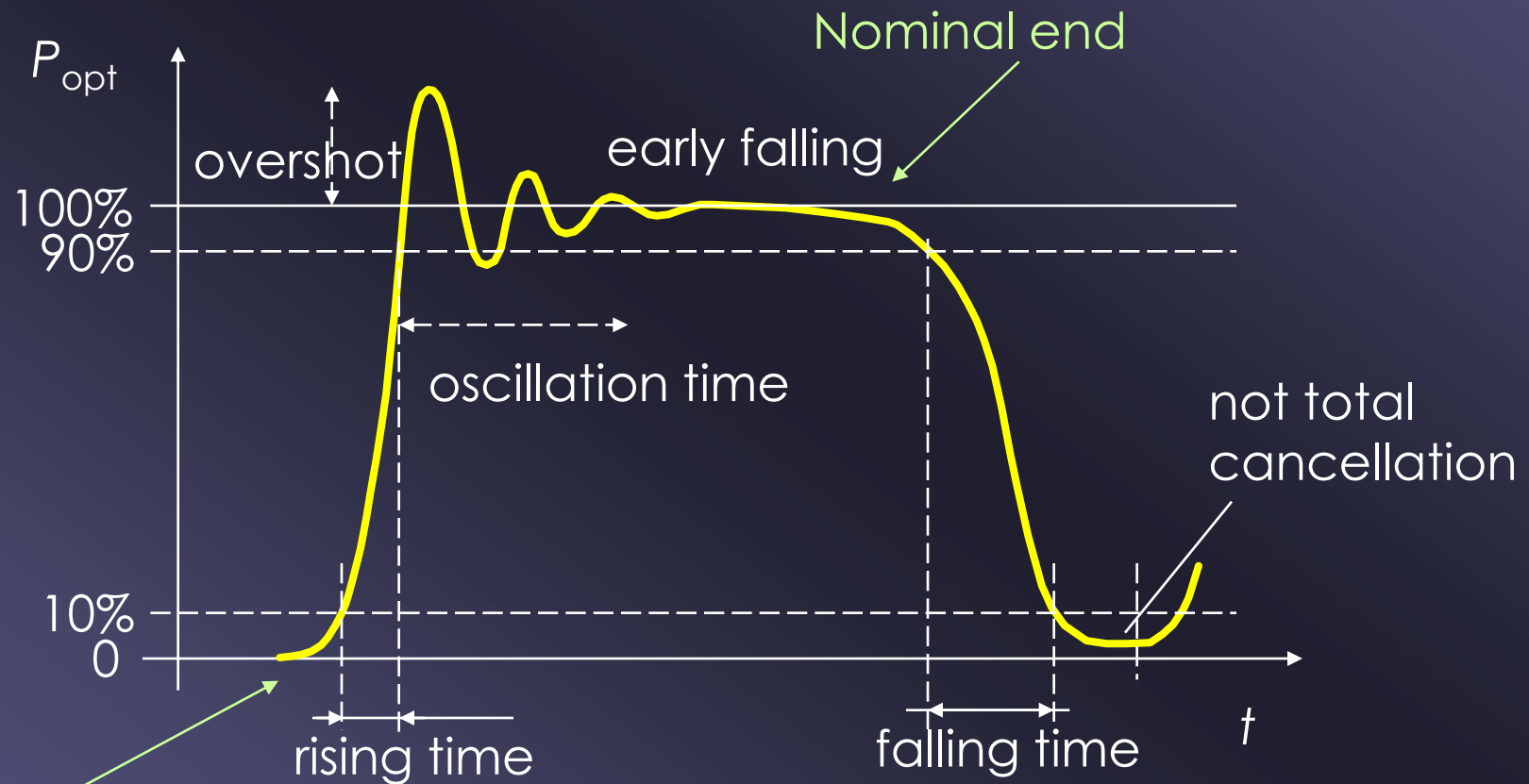
- Driving current is modulated



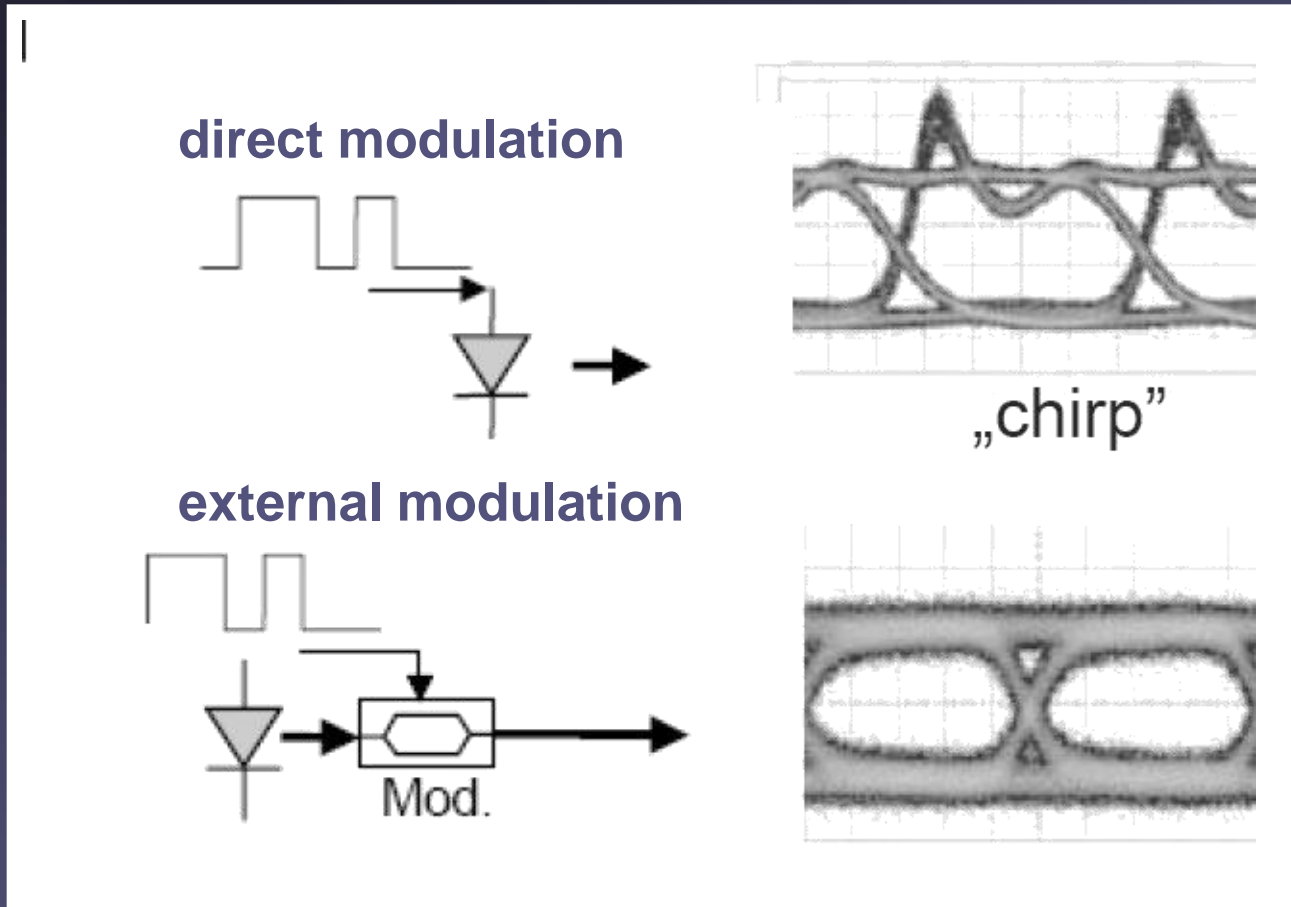
• Laser driver



Distortion of the pulse



Direct vs. external modulation



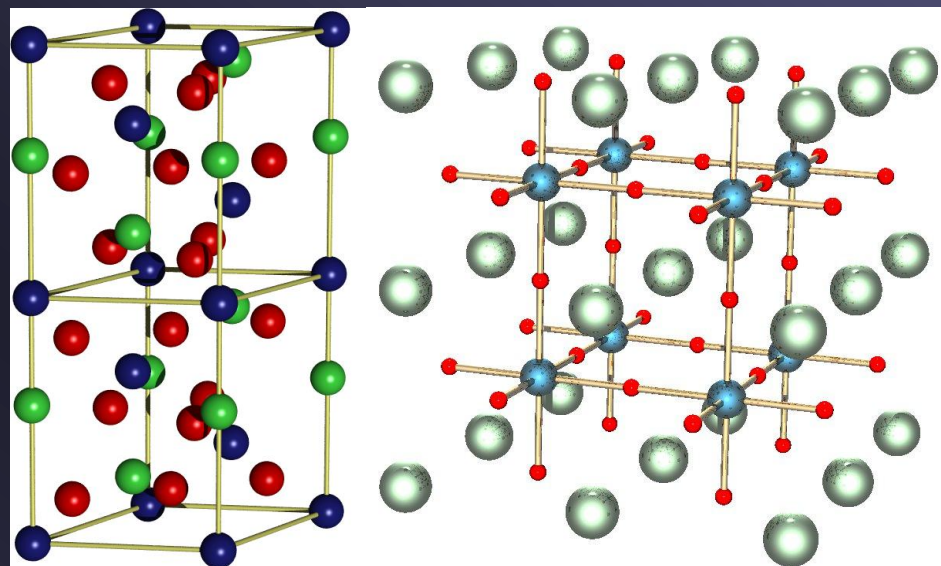
Optical property changes due to electronical field changes

Types

- index change
 - linear – Pockels
 - quadratic – Kerr
- activity change – electrogyration
- absorption change – electroabsorption
- gap change – Frank-Keldysh (bulk semiconductors)
– quantum confined Stark (q-wells)
- liquid crystals

Electrooptical materials

- LiNbO_3
- BaTiO_3



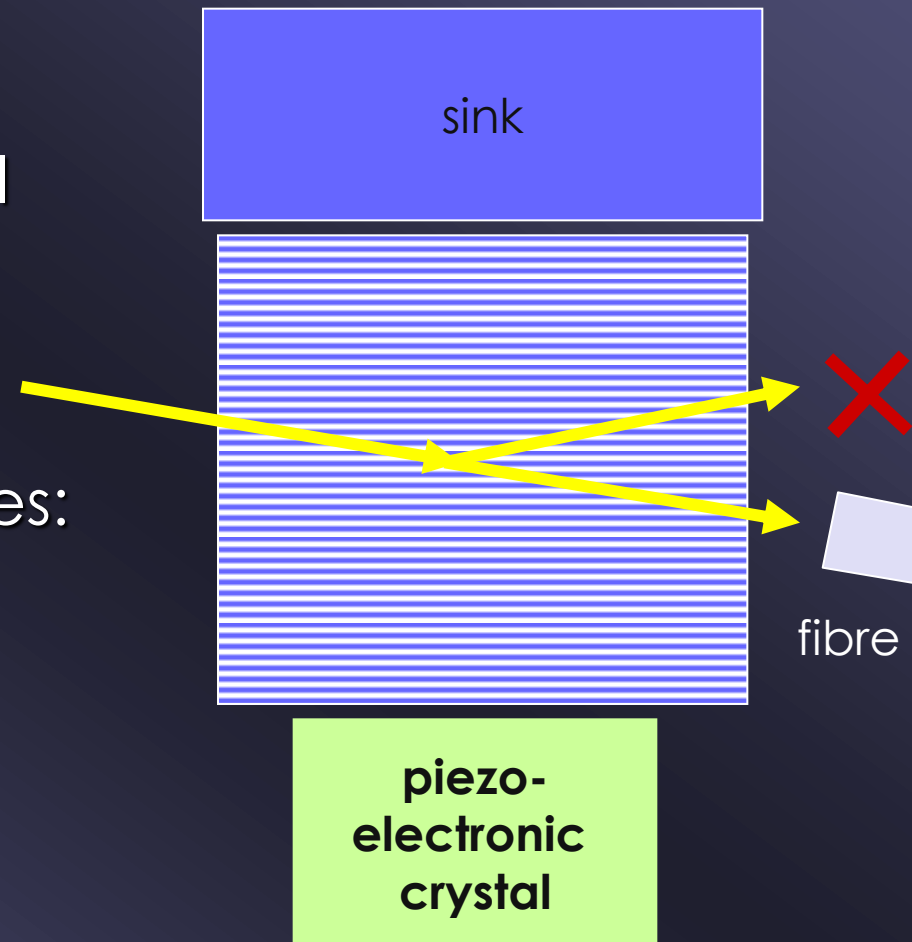
By Cadmium at English Wikipedia - Transferred from en.wikipedia to Commons., Public Domain, <https://commons.wikimedia.org/w/index.php?curid=2527511>, By Ahellwig - created with Povray 3.6, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=163749>

Optical property changes due to magnetic field changes

- index change
- Faraday rotator
- CdMnTe, CdMnHgTe, TdGdG, ...

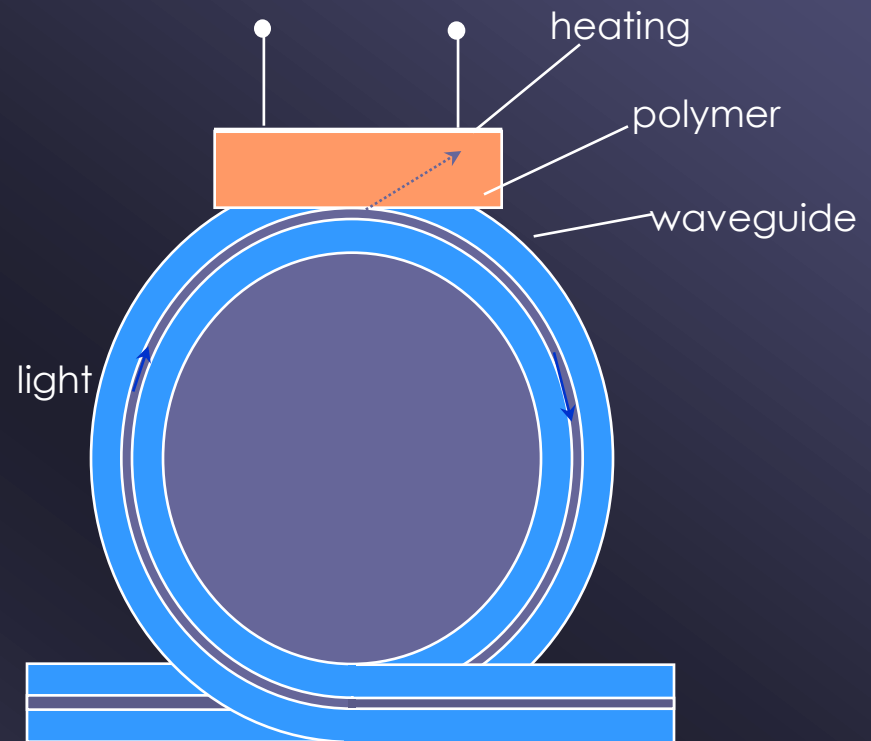
Optical property changes due to density or strain changes

- index change
 - a piezoelectronic signal transceiver generates acoustic waves in the crystal,
 - due to the density waves: **optical grating**
 - light is reflected on the grating
- material, e.g.



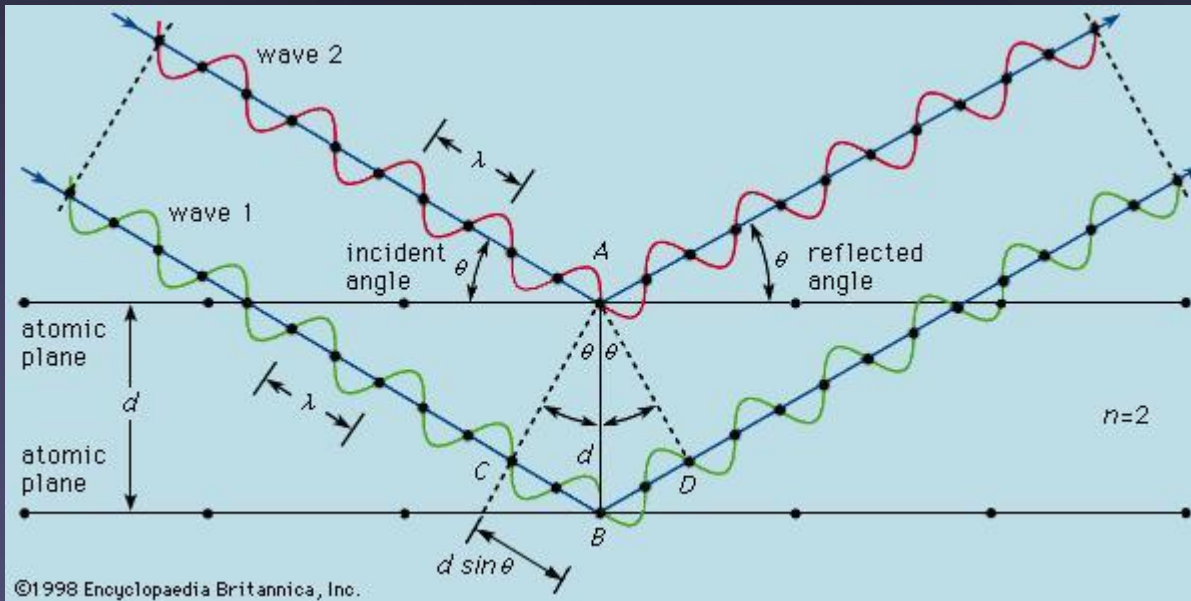
Optical property changes due to temperature changes

- index change



Periodically changing optical properties can induce constructive or destructive interference

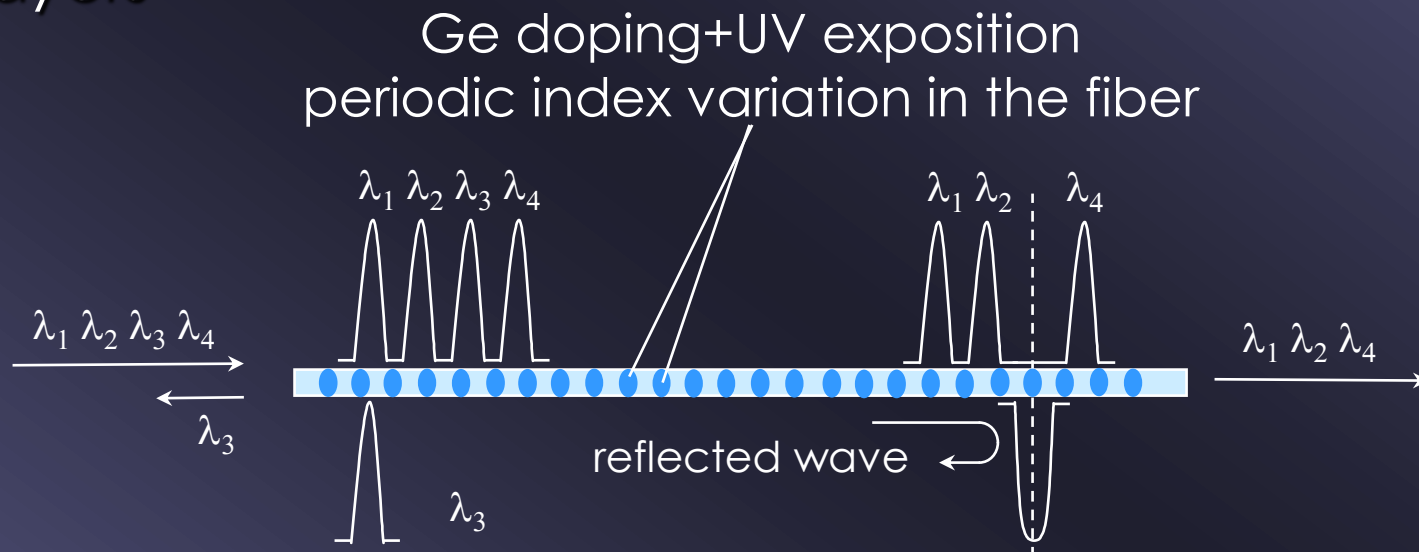
- gratings
- multilayers



www.britannica.com

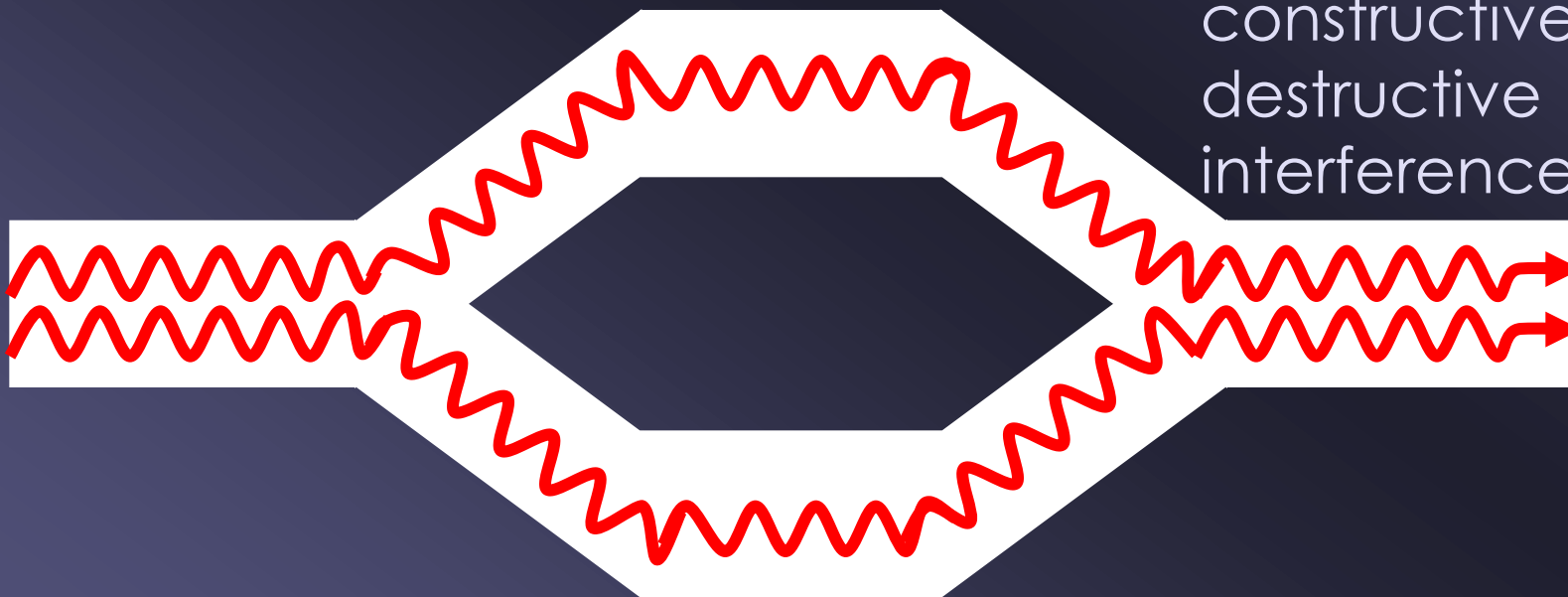
Periodically changing optical properties can induce constructive or destructive interference

- gratings
- multilayers



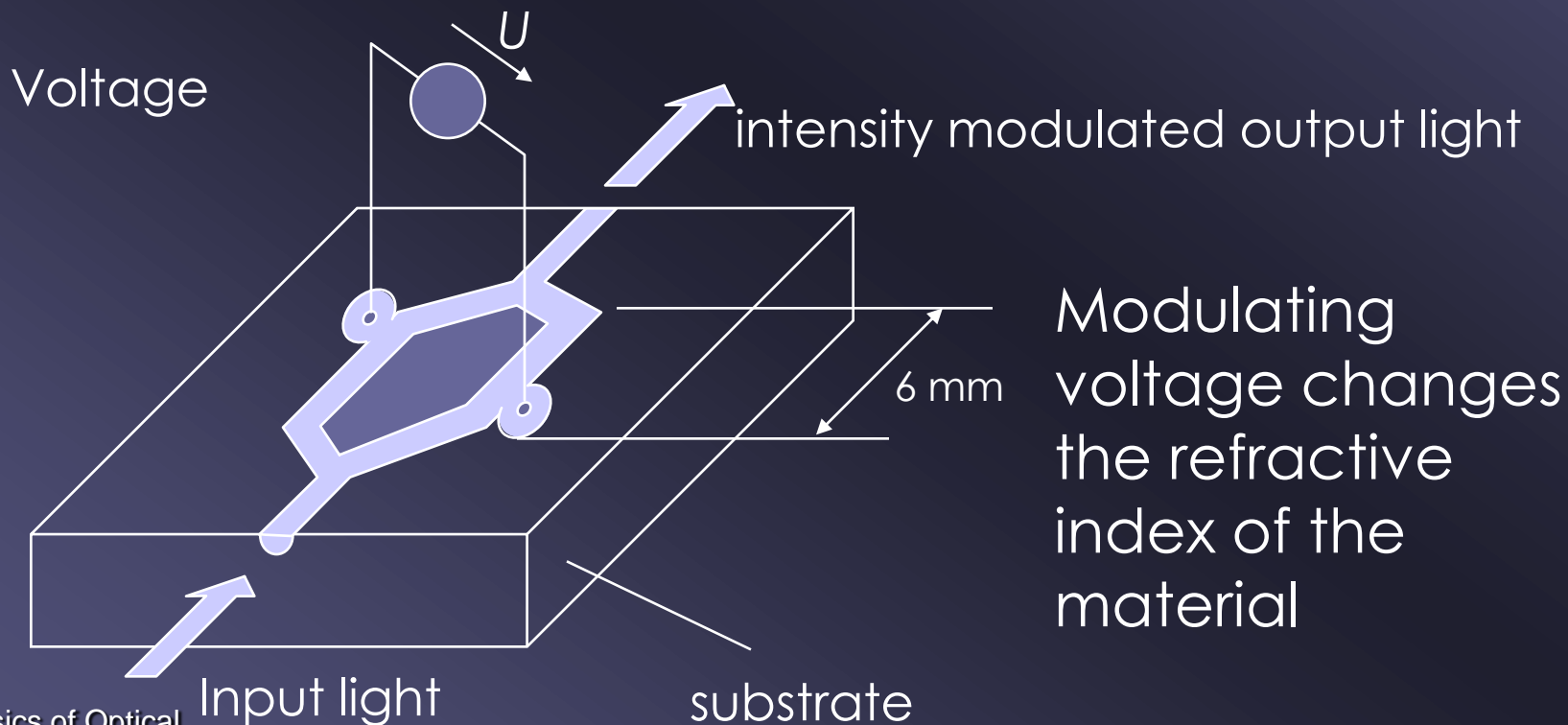
Bragg grating in fibers – add and drop MUX sensors

- External modulation with Mach—Zehnder interferometer



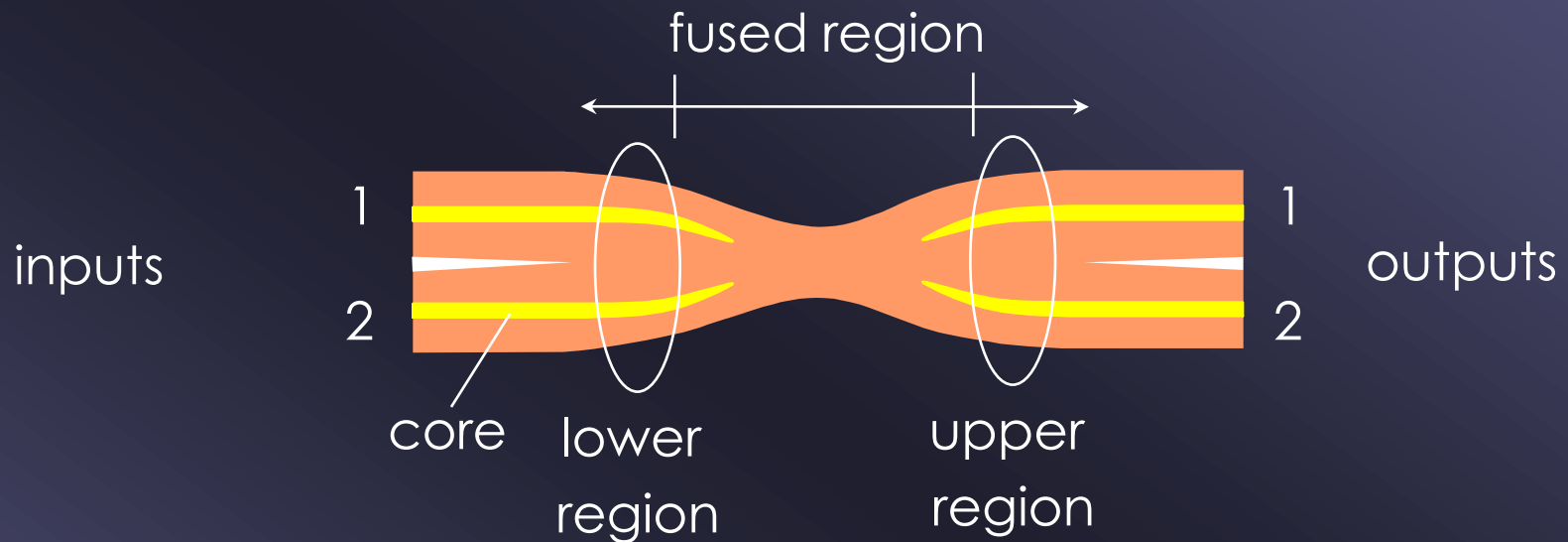
Difference in optical paths → constructive or destructive interference

- External modulation with Mach—Zehnder interferometer



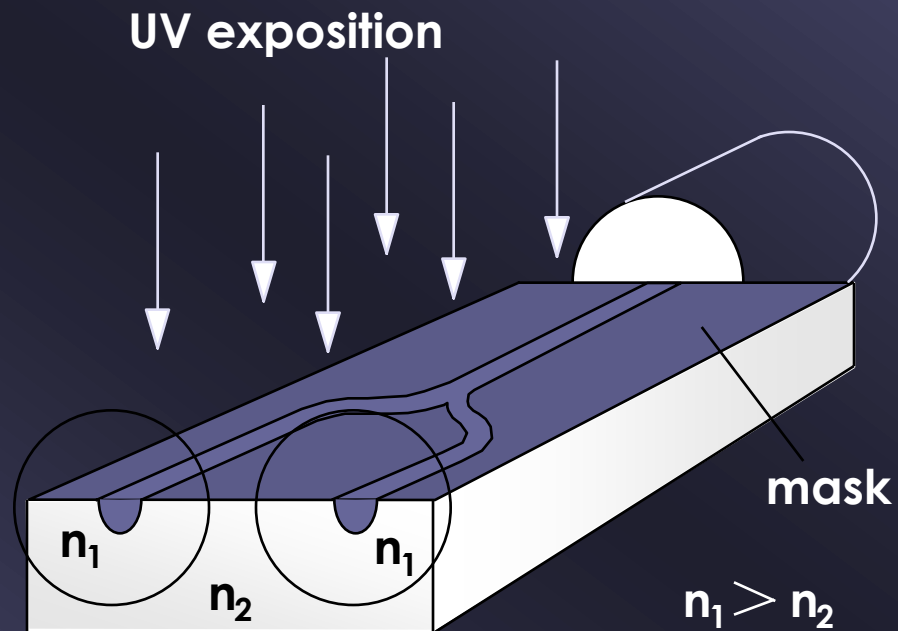
- Splitting
 - polymerized channels
 - fused fibers
 - interferometers
- Filtering, multiplexing, demultiplexing
 - prism
 - grating
 - Bragg layers

Fused fiber couplers



- splitting rate can be influenced by the fabrication process,
- coupling length can influence the wavelength selectivity

Polymerized optocoupler

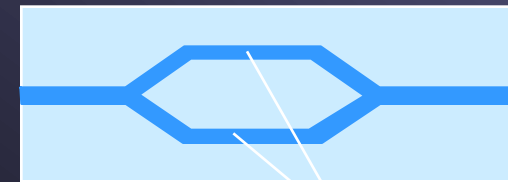


Optical slicing

(Optical slicing/interleaving)



Mach-Zender interferometer



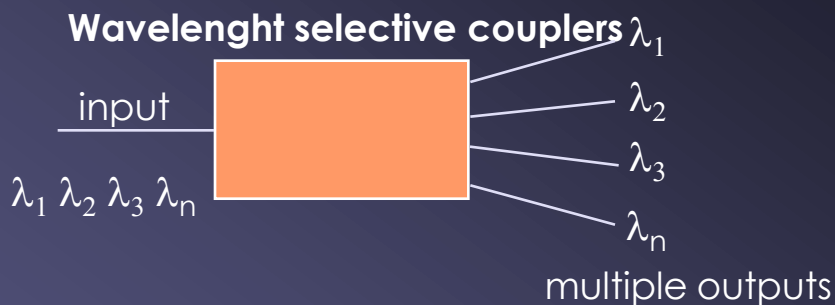
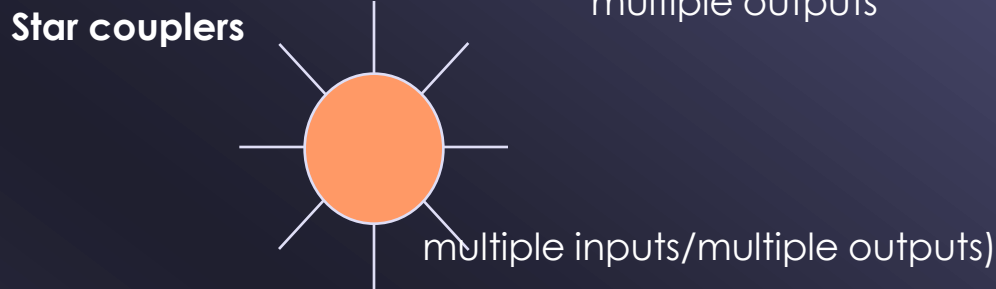
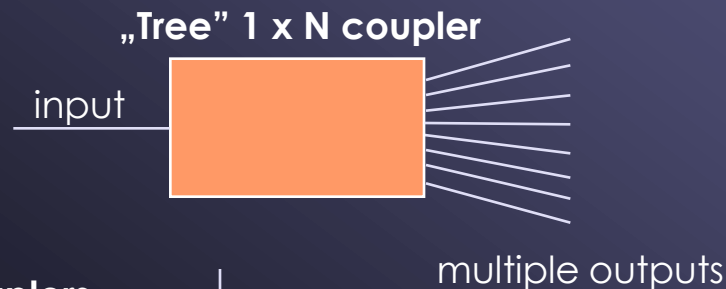
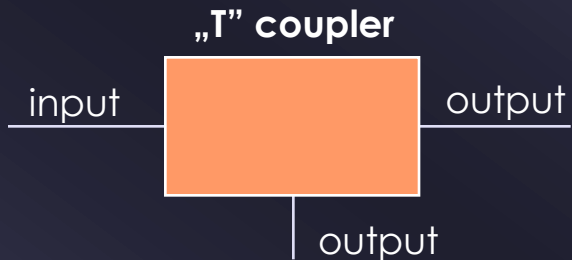
$\lambda/2$ path difference



(Fused Biconic Tapered)

Temperature dependence: 1 pm/°C (-5 °C ...+70 °C)

Optocoupler geometries



$$\text{Loss (dB)} = -10 \log \left[\frac{P_1 + P_2 + \dots + P_n}{P_{be}} \right]$$

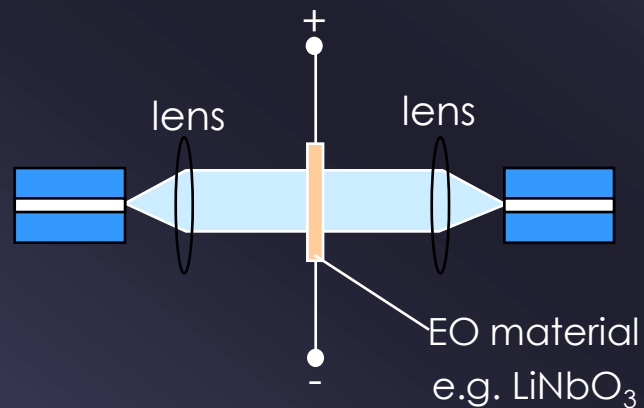
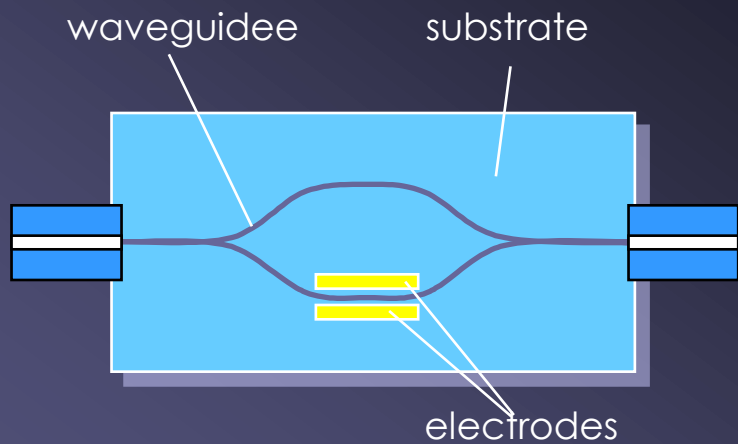
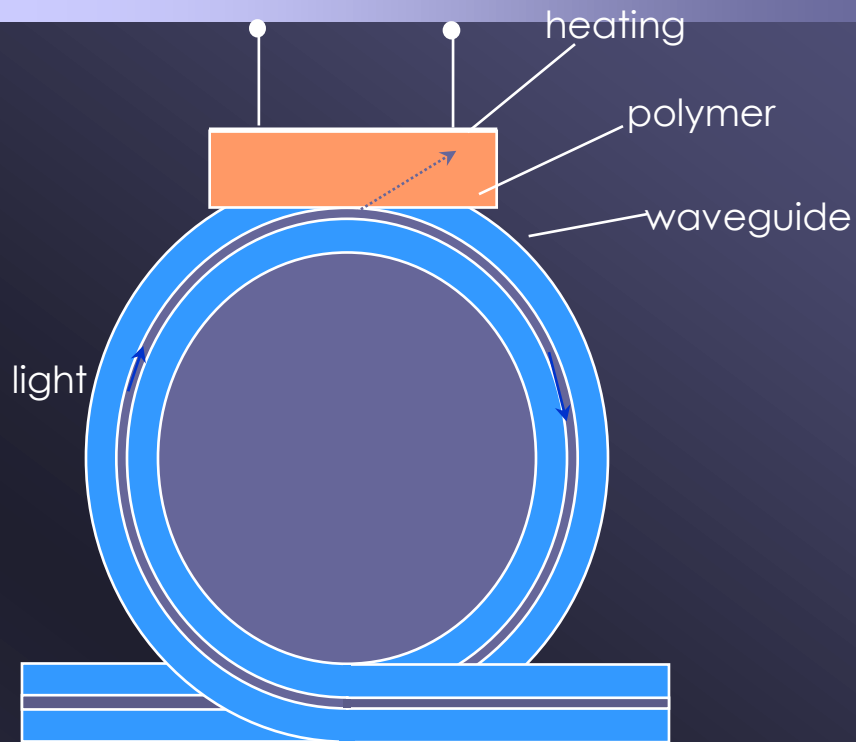
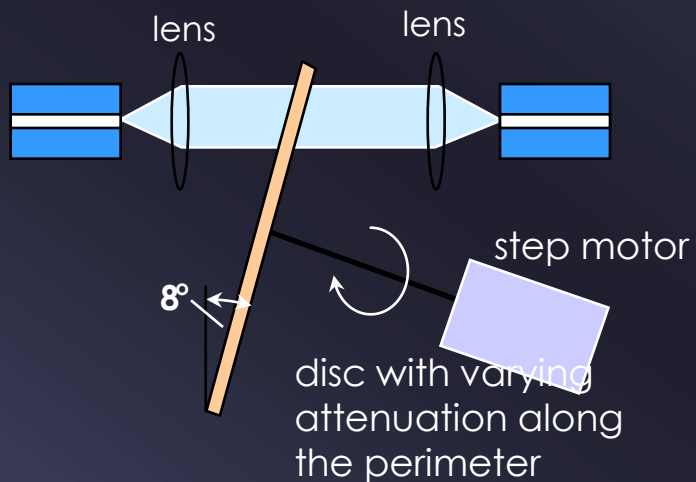
$$\text{Directivity (dB)} = -10 \log \left[\frac{P_n}{P_m} \right]$$

Types:

- active
- passzív
 - fix
 - variable
 - calibrated
 - not calibrated

Use: measurements, signal power control
in PONs

Különböző kivitelű csillapítók



For changing the optical path at the network nodes

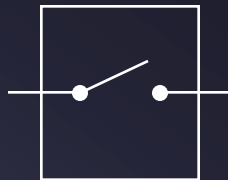
Applications:

- switching the signal path
- backward signal suppression
- multiplexing
- reserving optical paths
- measurements

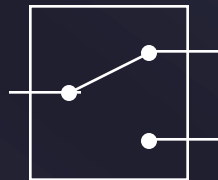
Types:

- electromechanical,
- electrooptical

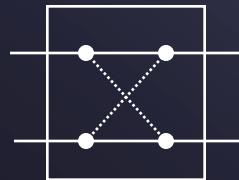
Optical switch topologies



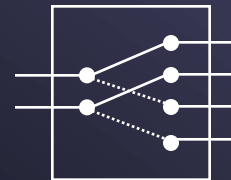
1 : 1



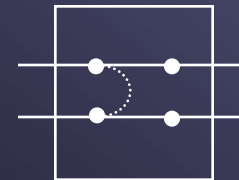
1 : 2



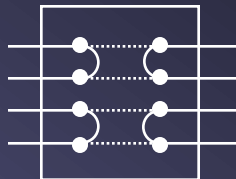
2 : 2



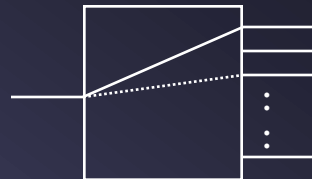
2 : 4



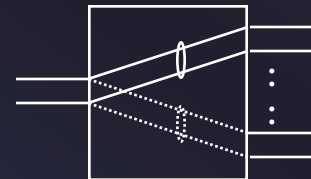
2:2 Bypass



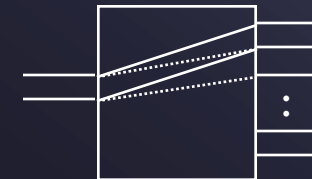
Dual reversing



1 x N



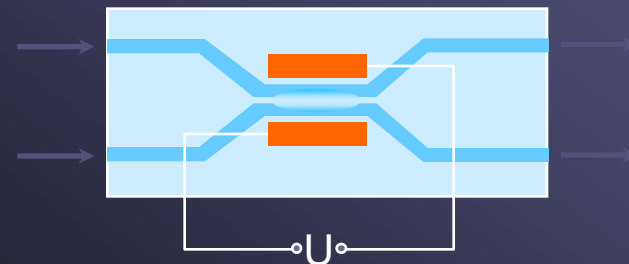
2 x (1xN)



2 x N

| | |
|-------------------------------|-----------------------|
| Insertion loss | 0.5 dB |
| Loss increment for repetition | 0.01 dB |
| Switching time | < 15 ns |
| Voltage, current | 5 V, 50 mA |
| Reflection attenuation | - 65 dB |
| Operation domain | 1300, 1550, (1650) nm |

Electrooptical switches



Advantages:

ns switching times,
stable

Disadvantages

large insertion loss,
large PDL, crosstalk
no favoured state

substrate: lithium-niobate
barium-titanate
elektrodes: Si-Mg oxide

Movable elements: miniature mirrors or prisms

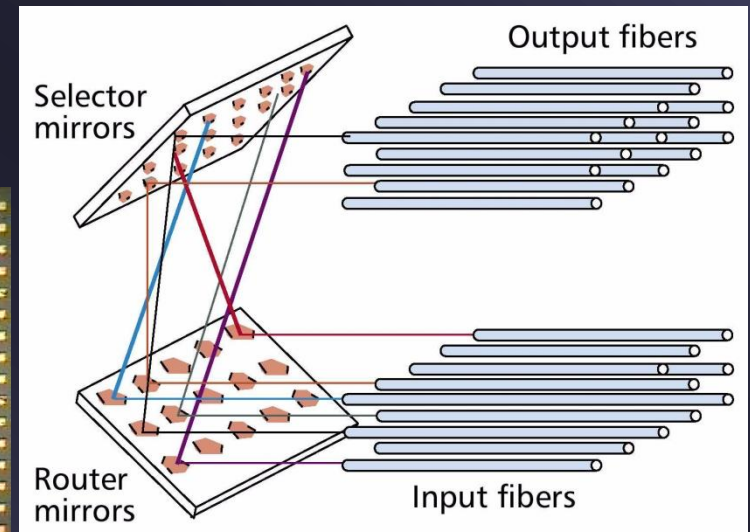
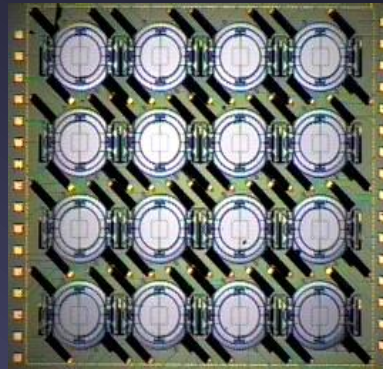
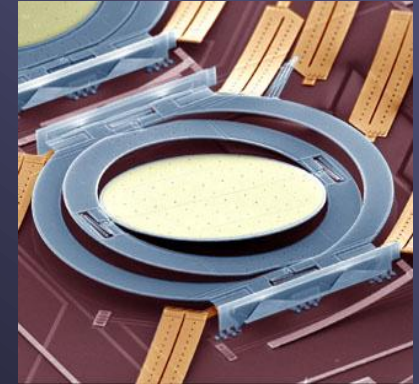
Moving elements: solenoids, piezos

Advantages:

- low polarization and wavelength dependence,
- insensitive for environmental effects,
- low power control signals,
- cheap manufacturing

Disadvantages:

- complicated control for larger switch matrices,
- ms switching times



Movable elements: miniature mirrors or prisms

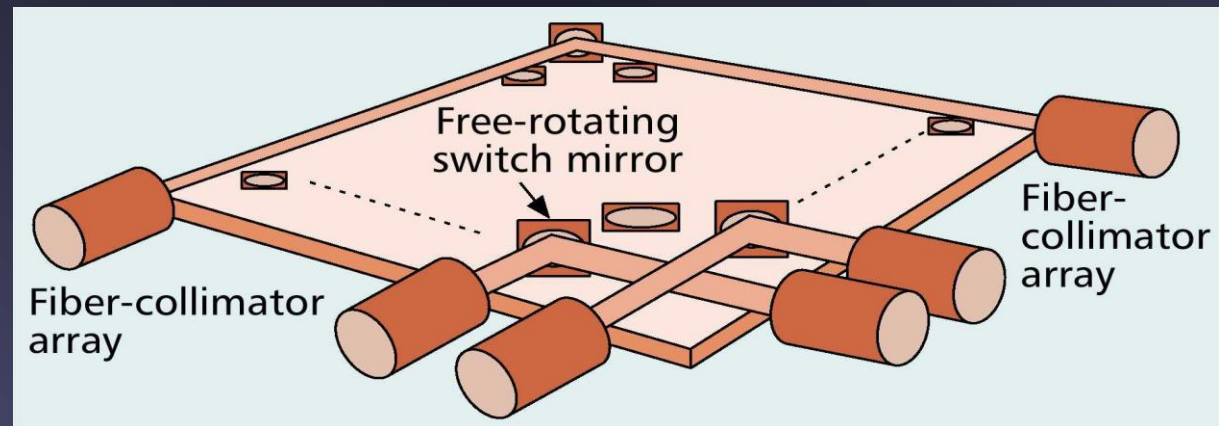
Moving elements: solenoids, piezos

Advantages:

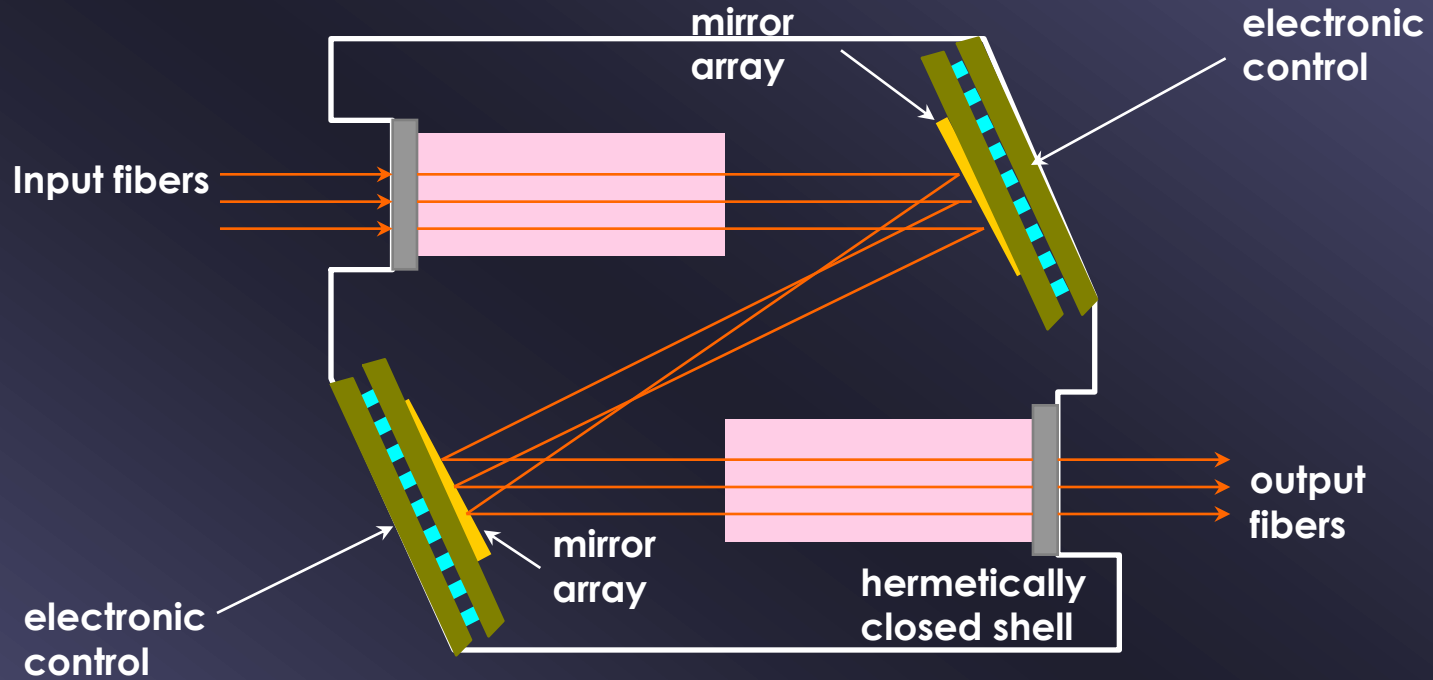
- low polarization and wavelength dependence,
- insensitive for environmental effects,
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Disadvantages:

- complicated control for larger switch matrices,
- ms switching times



OXC switching cell



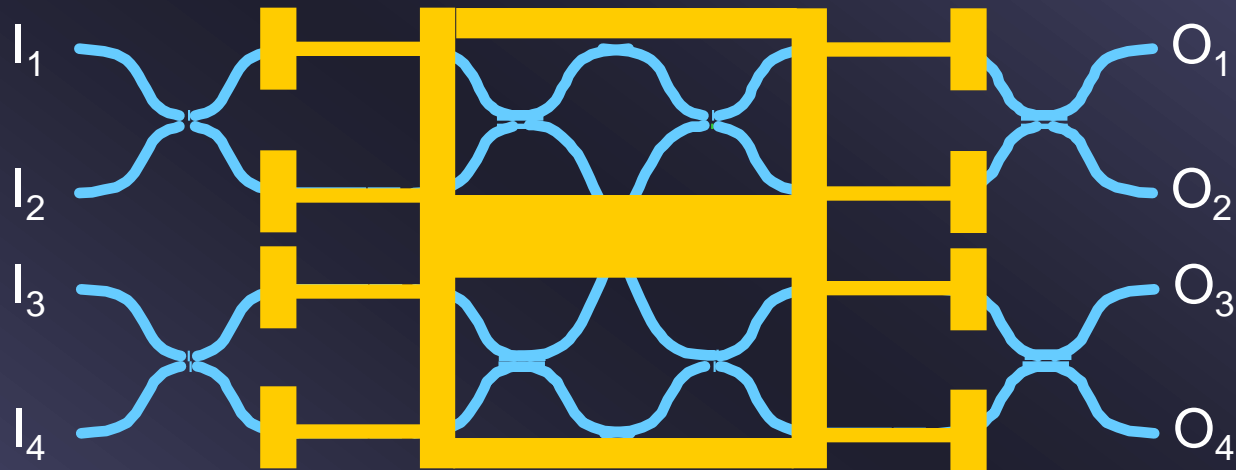
Thermally tunable polymer waveguides

- temperature dependent index,
- silitium substrate
- heating: thin film electrode on the polymer stack

Properties:

- acceptable attenuation,
- medium polarization dependence,
- large crosstalk,
- high power consumption,
- switching times: 1 ... 10 ms

2x2 thermo-optical switch array

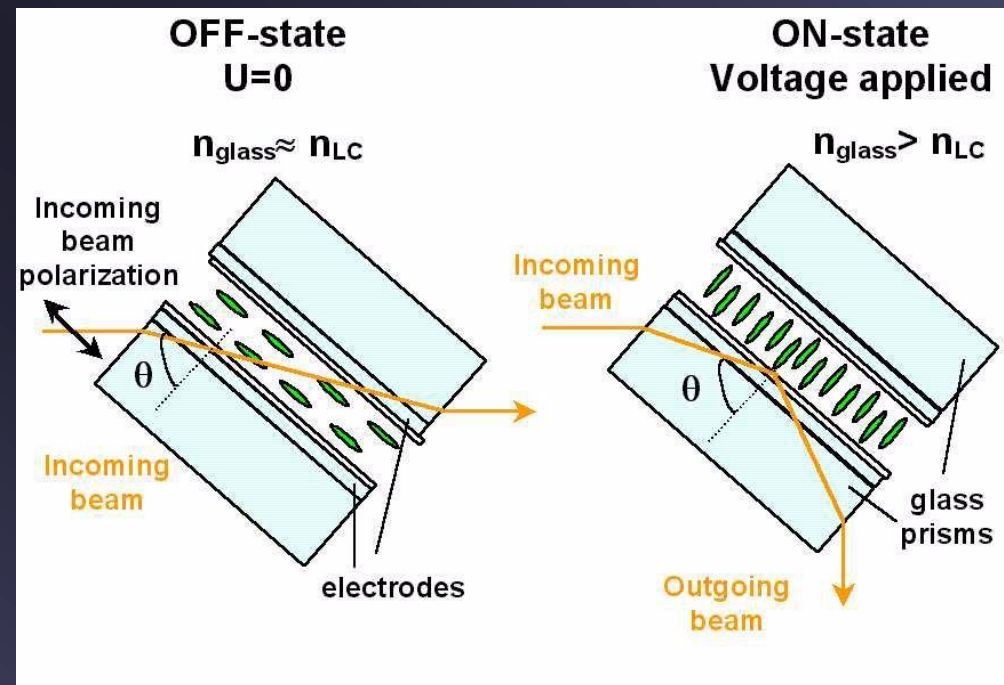


Liquid crystal cells

- polarization splitters
- voltage controlled polarization
- polarization sensitive or insensitive switch arrays

Properties

- large attenuation,
- higher crosstalk,
- complicated control,
- switching time:
 - 100 ms nematic liquids,
 - 10 ms ferroelectric liquids



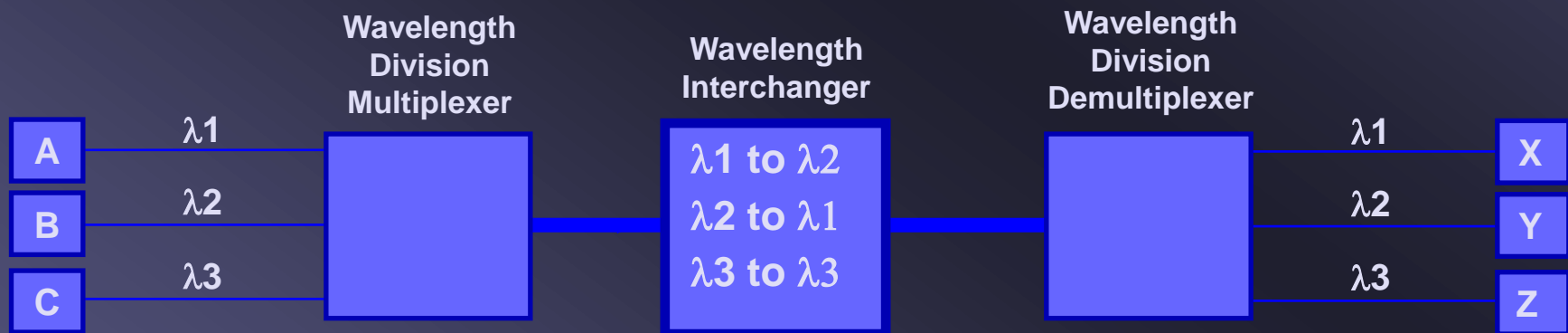
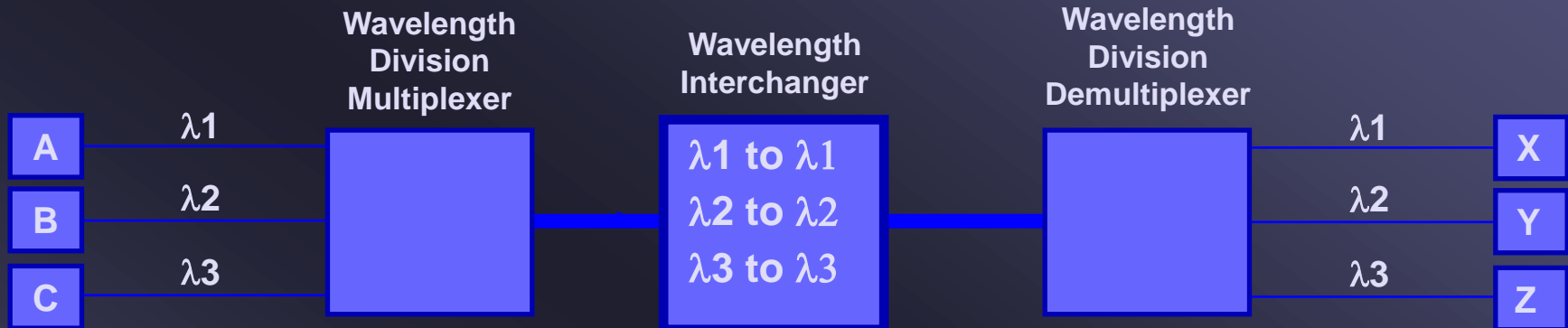
Directed acoustical waves influence the optical medium

- material e.g., TeO_2 ,
- index changes due to transversal acoustic waves,
- tunable by frequency

Properties:

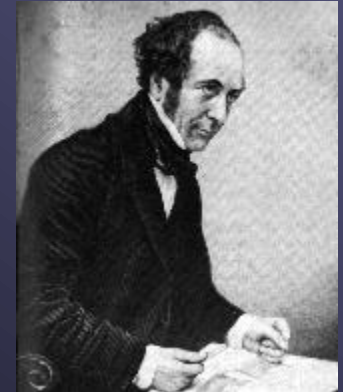
- wavelength dependent attenuation,
- costly control circuits,
- switchin time: ~ 10 ms,
- low density



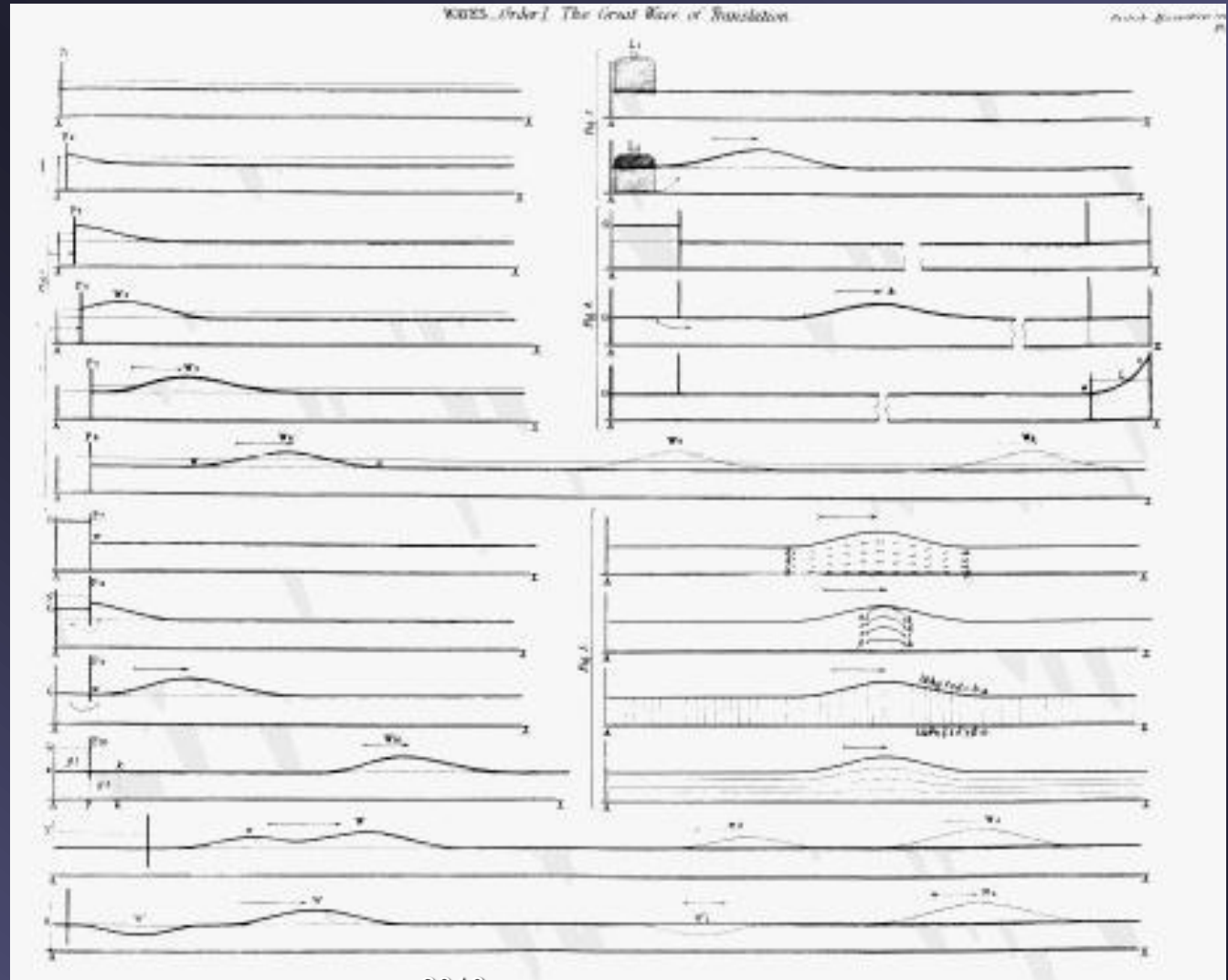


- Nonlinear effects in fibers
- History of solitons
- Korteweg—deVries equations
- Envelop solitons
- Solitons in optical fibers
- Amplification of solitons – optical soliton transmission systems

- John Scott Russel (1808-1882)
- 1834, Union Canal, Hermiston near Edinburgh, a boat was pulled
- after the stop of the boat a „wave of translation” arised
- 8-9miles/hour wave velocity
- traveled 1-2 miles long



History of solitons



J. S. Russel,
 Report on Waves,
 1844

History of solitons





Scott Russel Aqueduct,
1995

Heriot-Watt University
Edinburgh



- 1870s J. Boussinesq, Rayleigh both deduced the secret of Russel's waves: the dispersion and the nonlinearity cancels each other
- 1964 Zabusky and Kruskal solves the KdV equation numerically, solitary wave solutions: **soliton**
- 1960s: nonlinear wave propagation studied with computers: many fields were found where solitons appear

- 1970s A. Hasegawa proposed solitons in optical fibers
- 1980 Mollenauer demonstrated soliton transmission in optical fiber (10 ps, 1.5 μm , 700 m fiber)
- 1988 Mollenauer and Smith sent soliton light pulses in fiber for 6000 km without electronic amplifier

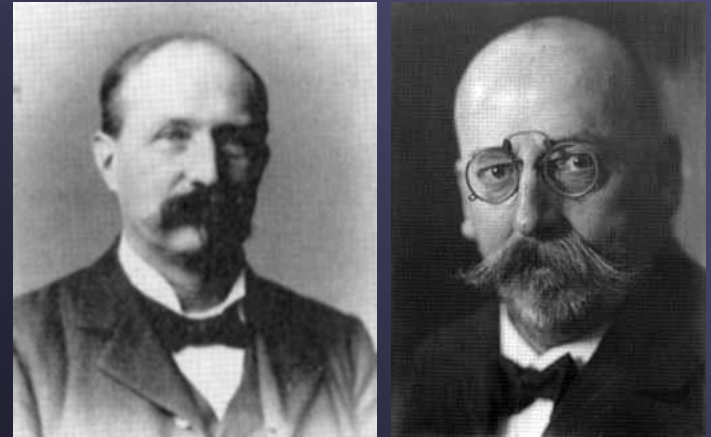
Korteweg—deVries equations

- In 1895 Korteweg and deVries modeled the wave motion on the surface of shallow water by the equation

$$\frac{\partial h}{\partial \tau} + h \frac{\partial h}{\partial \xi} + \frac{\partial^3 h}{\partial \xi^3} = 0$$

where h wave height
 τ time in coordinates
 ξ space coordinate

} moving with the wave



Derivation of the KdV equation

- a wave h propagating in x direction can be described in the coordinate system (ξ, τ) traveling with the wave as

$$\frac{\partial h}{\partial \tau} = 0$$

- Using the original (x, t) coordinates:

$$\frac{\partial h}{\partial \tau} = 0 \quad \xrightarrow[\substack{x = \xi + v\tau, \\ t = \tau}]{\text{}} \quad \frac{\partial h}{\partial t} + v \frac{\partial h}{\partial x} = 0$$

Stationary solution of the KdV equation

- Dispersive and nonlinear effects can balance to make a stationary solution

$$\frac{\partial h}{\partial \tau} + h \frac{\partial h}{\partial \xi} + \frac{\partial^3 h}{\partial \xi^3} = 0$$

$$v(h) = v_0 + \text{const} \cdot h$$

$$\begin{aligned} \omega &= kv = \omega_0 + \text{const} \cdot k^2 \omega_0 \\ &= \omega_0 + \text{const} \cdot k^3 v_0 \end{aligned}$$

Stationary solution of the KdV equation

- Dispersive and nonlinear effects can balance to make a stationary solution

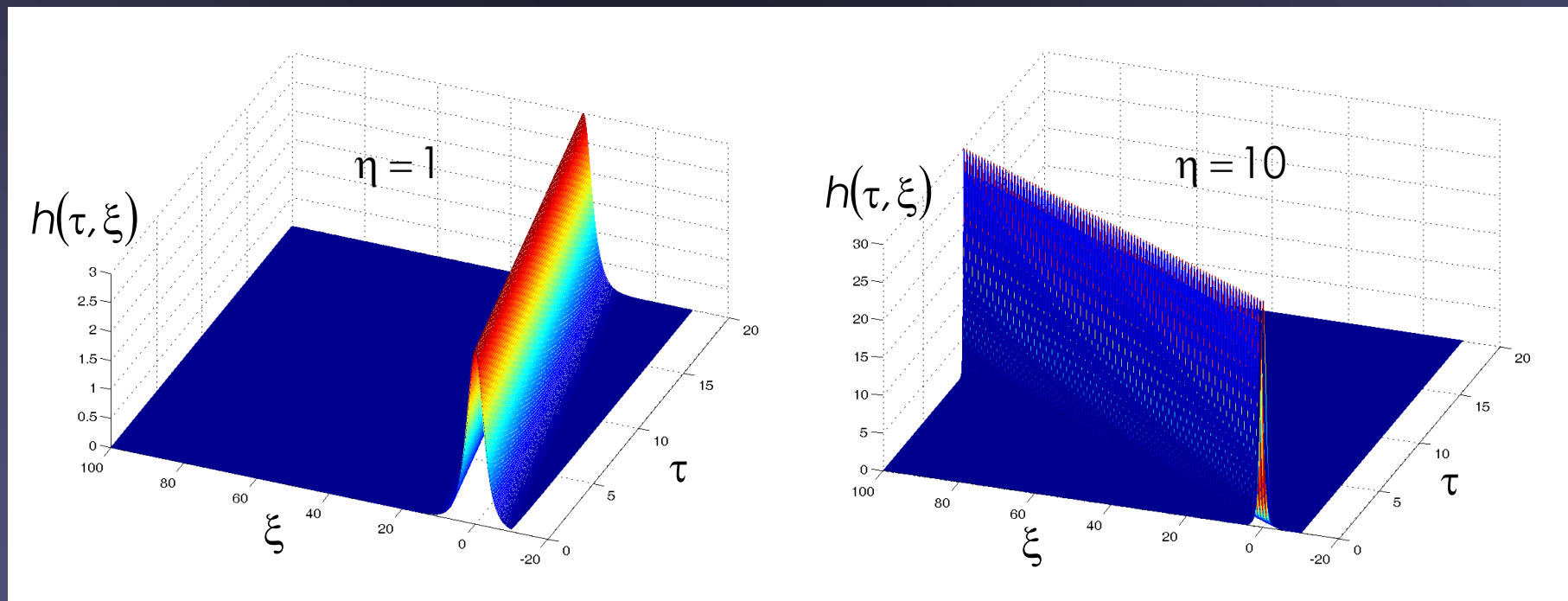
$$\frac{\partial h}{\partial \tau} + h \frac{\partial h}{\partial \xi} + \frac{\partial^3 h}{\partial \xi^3} = 0$$

$$h(\tau, \xi) = 3\eta \operatorname{sech}^2 \frac{\sqrt{\eta}}{2} (\xi - \eta\tau)$$

where η is the velocity of the solitary wave in the (ξ, τ) space

Stationary solution of the KdV equation

$$h(\tau, \xi) = 3\eta \operatorname{sech}^2 \frac{\sqrt{\eta}}{2} (\xi - \eta\tau)$$



The KdV equation and the inverse scattering problems

- the Schrödinger equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + (\lambda - u(x, t))\Phi = 0$$

- if „potential” $u(x, t)$ satisfies a KdV equation,
 - λ is independent of time
 - $u(x, 0) \rightarrow 0$ as $|x| \rightarrow \infty$
 - the Schrödinger equation can be solved for $t=0$ for a given initial $u(x, 0)$

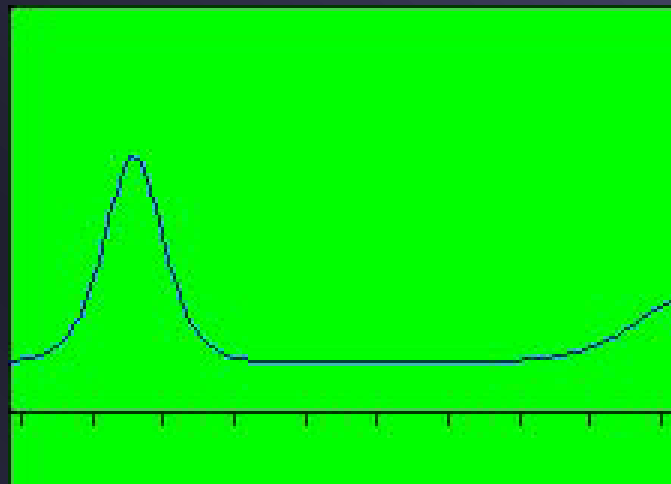
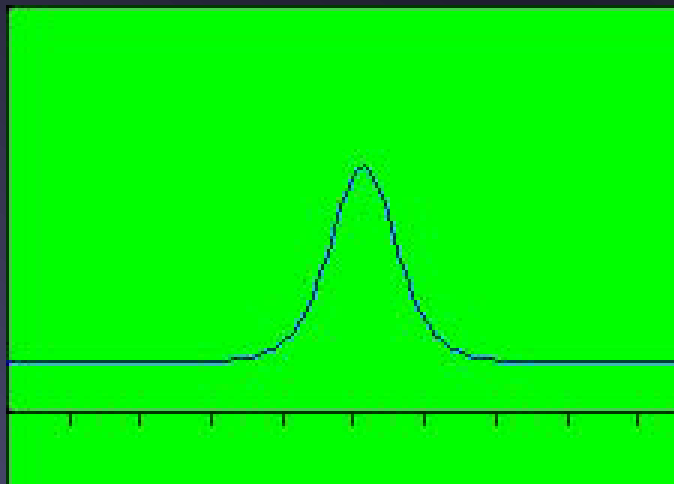
The KdV equation and the inverse scattering problems

- $t=0$ scattering data can be derived from the $t=0$ solution
- the time evolution of Φ and thus the scattering data is known

$$\frac{\partial \Phi}{\partial t} = A \frac{\partial^3 \Phi}{\partial x^3} + B \frac{\partial U}{\partial x} \Phi + C U \frac{\partial \Phi}{\partial x}$$

- $U(x,t)$ can be found for each $\Phi(x,t)$ by inverse scattering methods.

Solutions of KdV equations with various boundary conditions in various dimensions



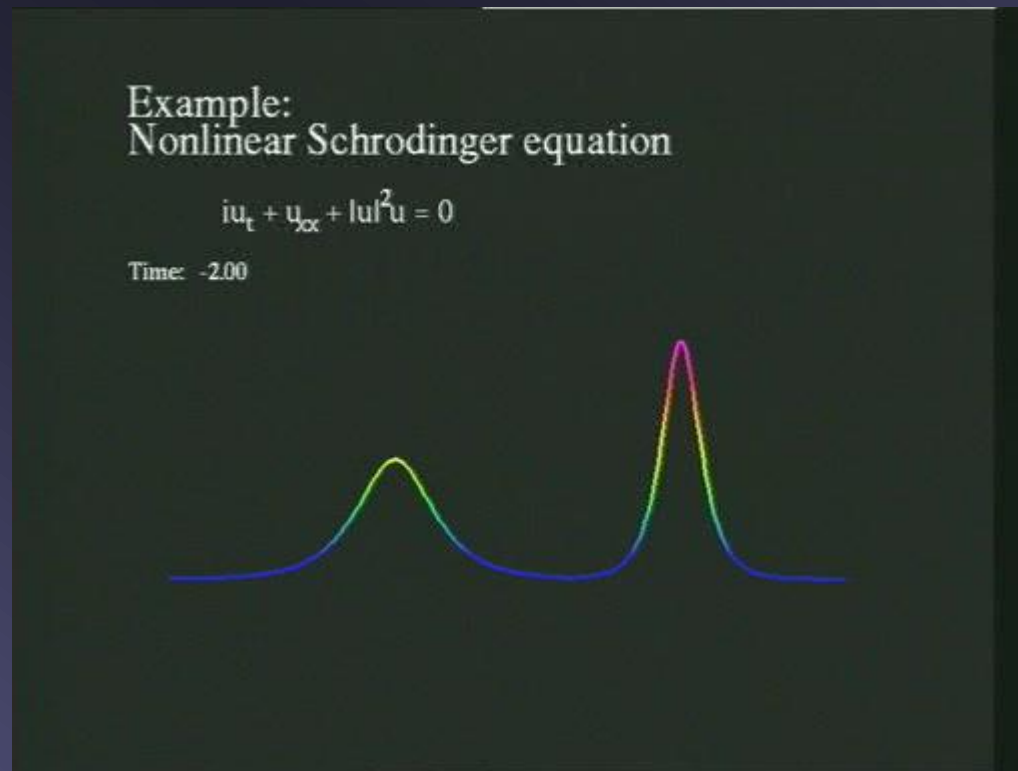
Solutions of KdV equations with various boundary conditions in various dimensions



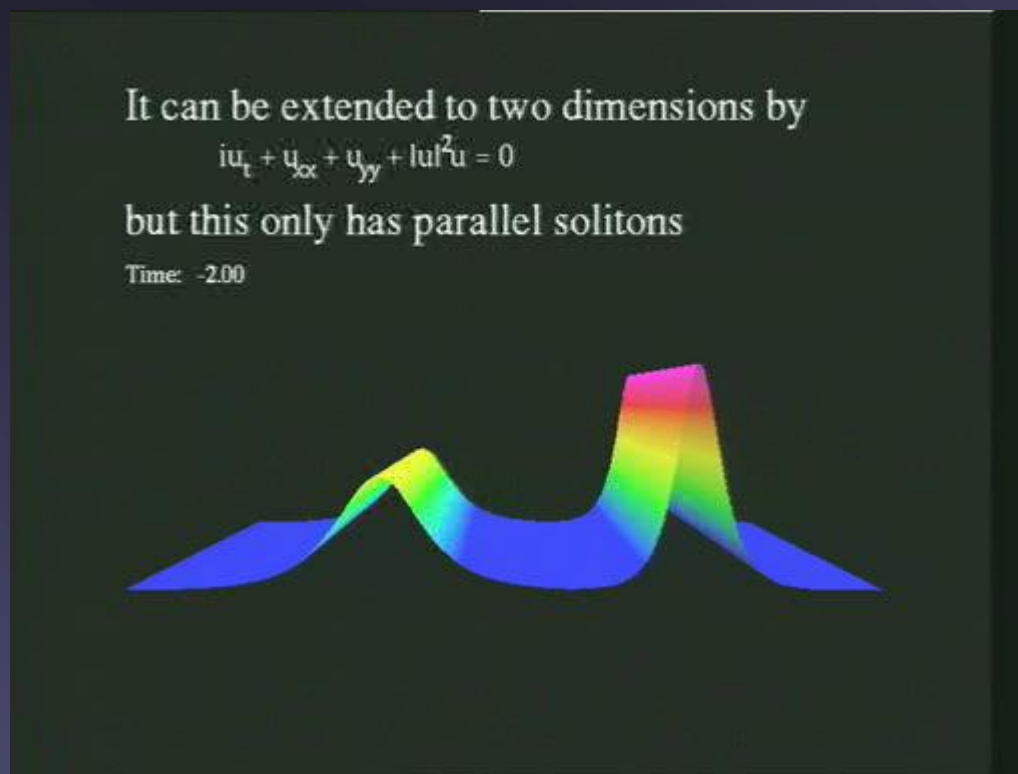
Solutions of KdV equations with various boundary conditions in various dimensions



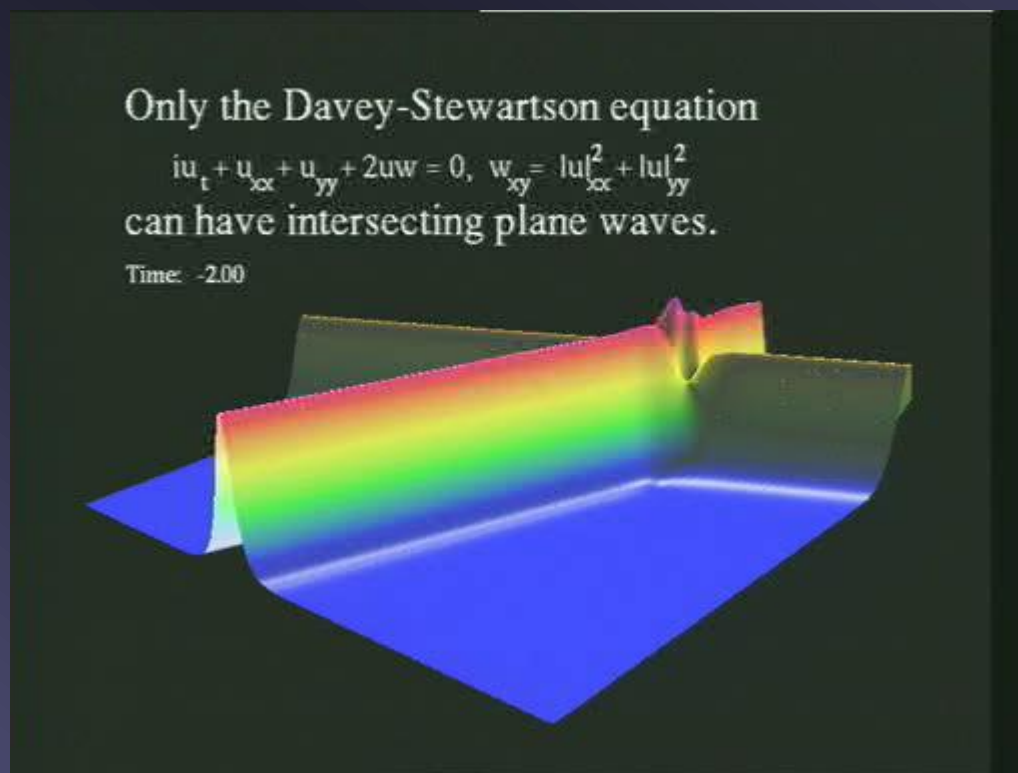
Solutions of KdV equations with various boundary conditions in various dimensions



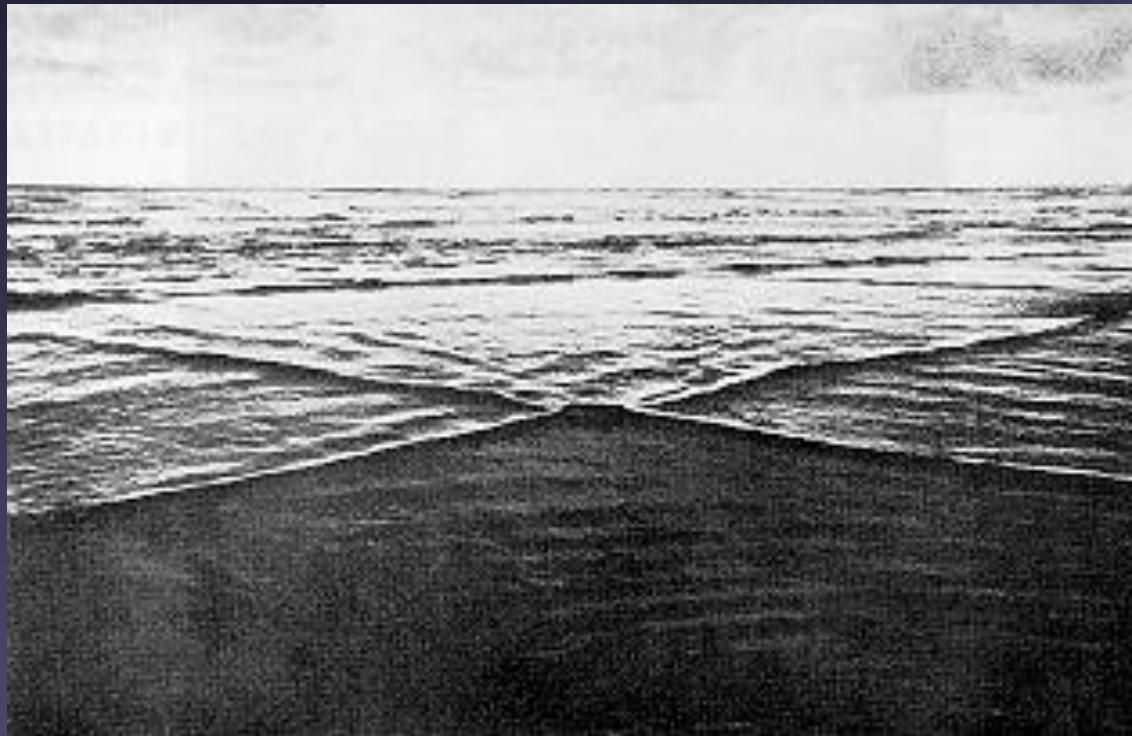
Solutions of KdV equations with various boundary conditions in various dimensions



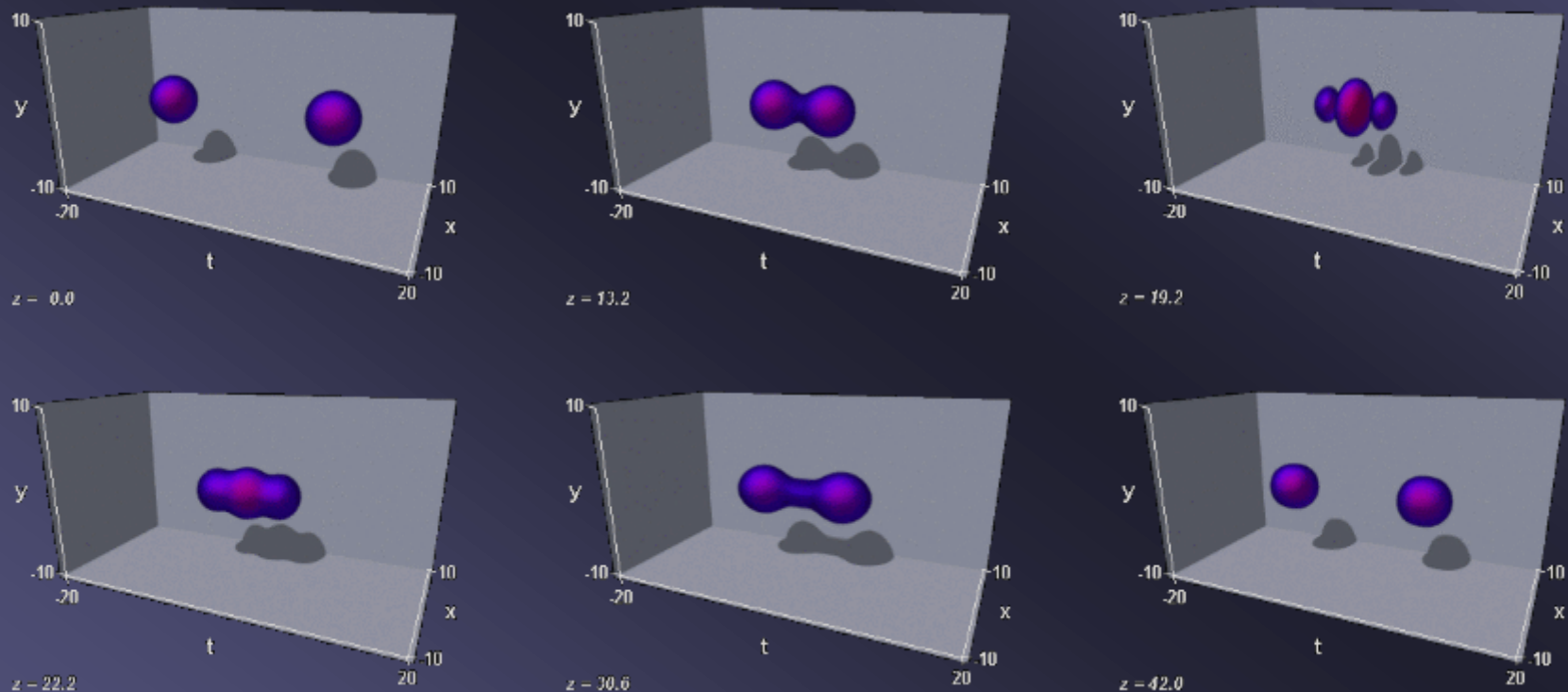
Solutions of KdV equations with various boundary conditions in various dimensions



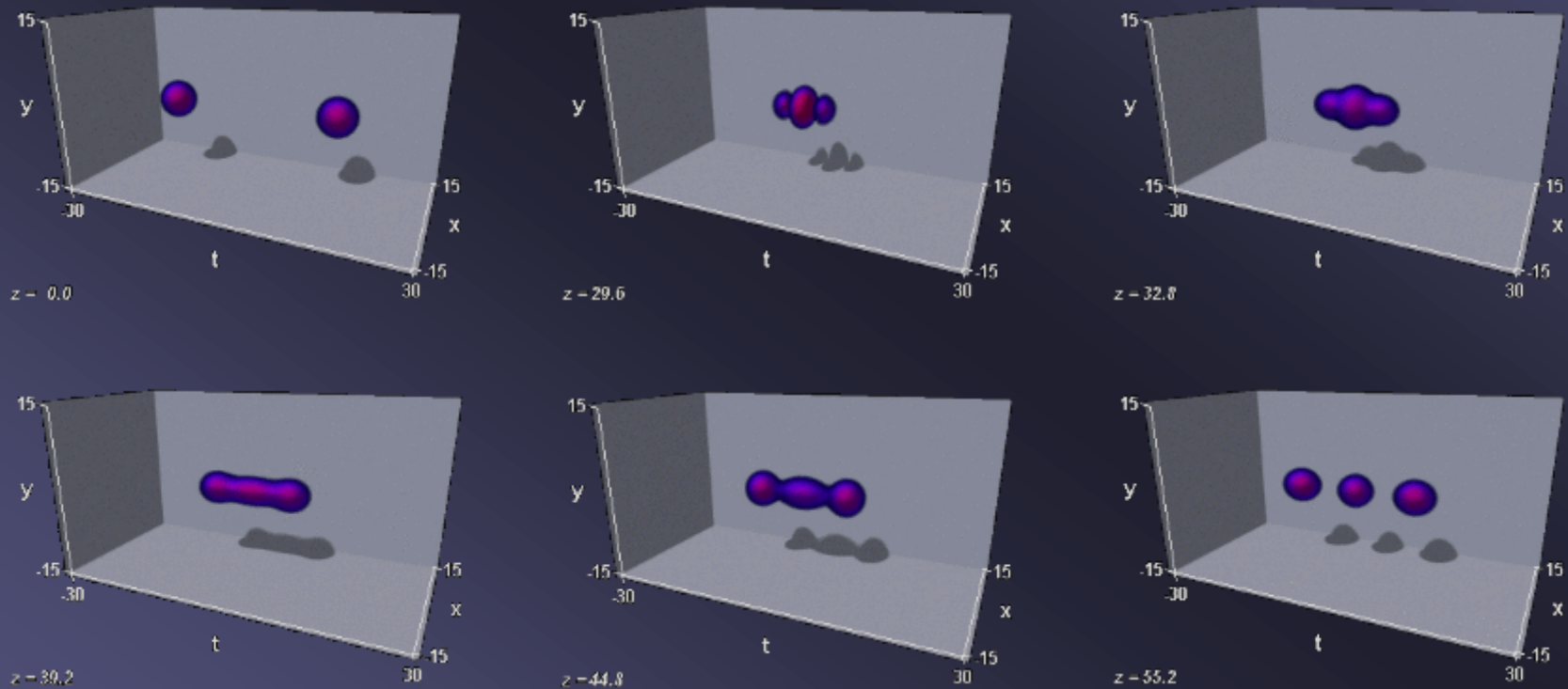
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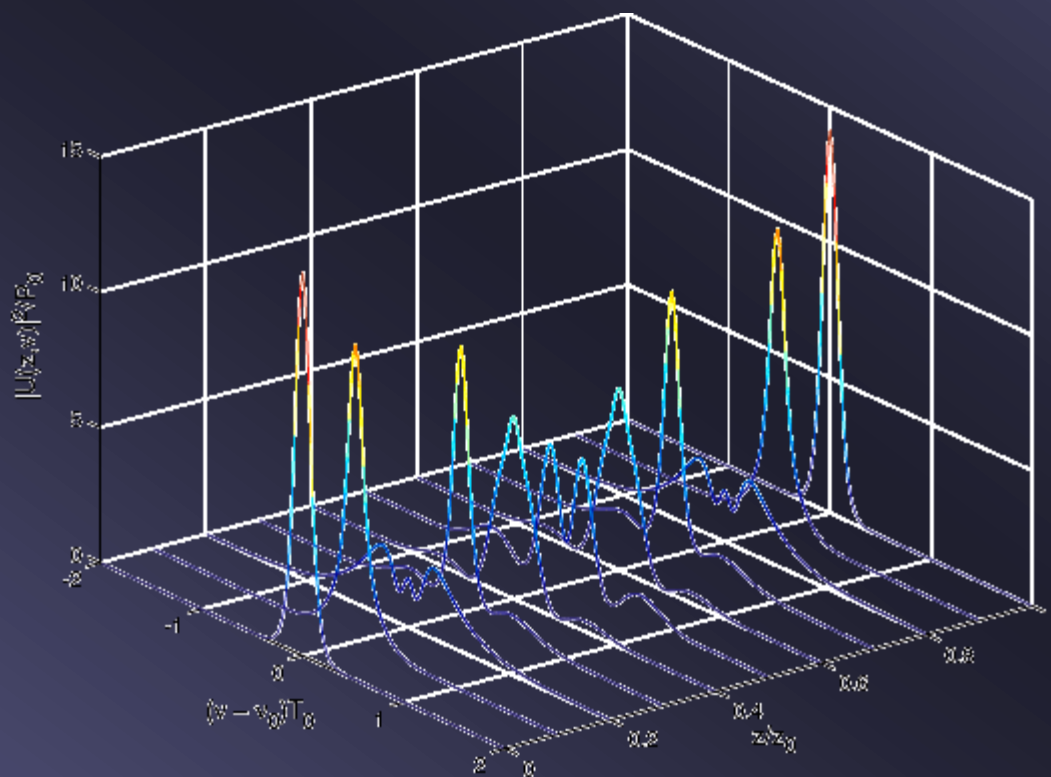
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Solutions of KdV equations with various boundary conditions in various dimensions



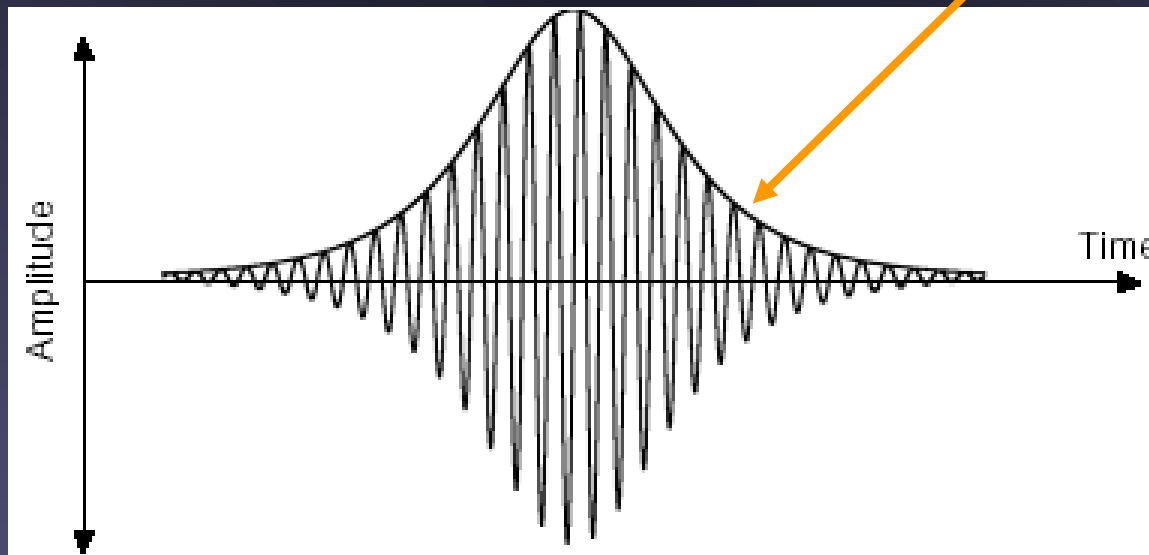
Solutions of KdV equations with various boundary conditions in various dimensions



higher
order
soliton

Envelop of a wave

- if the amplitude of a wave varies (slowly)



envelop
of the
wave

$$h(t, x)$$

complex
amplitude

- If the wave can be described by

$$E(x, t) = \text{Re}(\hat{E}(x, t) \cdot e^{i(k_0 x - \omega_0 t)})$$

the wave equation for the envelop $\hat{E}(x, t)$

$$i \frac{\partial \hat{E}}{\partial \xi} - \frac{k''}{2} \frac{\partial^2 \hat{E}}{\partial \tau^2} + g \frac{|\hat{E}|^2 \hat{E}}{\delta^2} = 0$$

reduction
factor, $\sim 1/2$

with $k'' = \left. \frac{\partial^2 k}{\partial \omega^2} \right|_{\omega=\omega_0} = D \frac{\lambda}{\omega}$, $\delta = \frac{\Delta \omega_0}{\omega_0}$ and $g = \frac{2\pi n_2 \alpha}{\lambda}$.

• Normalization

$$i \frac{\partial \hat{E}}{\partial \xi} - \frac{k''}{2} \frac{\partial^2 \hat{E}}{\partial \tau^2} + g \frac{|\hat{E}|^2 \hat{E}}{\delta^2} = 0$$

$$\begin{aligned}
 q &= \frac{\sqrt{g\lambda}}{\delta} \hat{E}, \\
 T &= \frac{\tau}{\sqrt{\lambda k''}}, \\
 X &= \frac{\xi}{\lambda}
 \end{aligned}$$

$$i \frac{\partial q}{\partial X} - \frac{1}{2} \frac{\partial^2 q}{\partial T^2} + |q|^2 q = 0$$

- Solving the non-linear Schrödinger equation

$$i \frac{\partial q}{\partial X} - \frac{1}{2} \frac{\partial^2 q}{\partial T^2} + |q|^2 q = 0$$

- test function

$$q(T, X) = \sqrt{\rho(T, X)} e^{i\sigma(T, X)}$$

- the new equation

$$i \frac{\partial \rho}{\partial X} + \frac{\partial}{\partial T} \left(\rho \frac{\partial \sigma}{\partial T} \right) = 0$$

- looking for solitary wave solution of the new equation

$$i \frac{\partial \rho}{\partial X} + \frac{\partial}{\partial T} \left(\rho \frac{\partial \sigma}{\partial T} \right) = 0$$

- if $|q|^2 = \rho$ is a stationary solution

$$\frac{\partial \rho}{\partial X} = 0 \quad \longrightarrow \quad \rho \frac{\partial \sigma}{\partial T} = C(X)$$

it can be shown, that C is independent of X

- the solutions

$$\rho = \rho_0 \operatorname{sech}^2 \sqrt{\rho_0} T$$

$$\sigma = \frac{\rho_0}{2}$$

- which give

$$q(T, X) = \eta \operatorname{sech}(\eta(T + \kappa X - \vartheta_0)) e^{-i \left(\kappa T - \frac{\eta^2 + \kappa^2}{2} X + \sigma_0 \right)}$$

ϑ_0 and σ_0 are phase constants

$\eta = \rho^{1/2}$:
amplitude and pulse width

κ :
transmission speed

- envelop equation of a light wave in a fiber

$$i \frac{\partial \hat{E}}{\partial \xi} - \frac{k''}{2} \frac{\partial^2 \hat{E}}{\partial \tau^2} + g \frac{|\hat{E}|^2 \hat{E}}{\delta^2} = 0$$

- fiber loss rate per unit length: γ

$$i \frac{\partial \hat{E}}{\partial \xi} - \frac{k''}{2} \frac{\partial^2 \hat{E}}{\partial \tau^2} + g \frac{|\hat{E}|^2 \hat{E}}{\delta^2} = -\frac{i\gamma \hat{E}}{\delta^2}$$

with

$$k'' = \left. \frac{\partial^2 k}{\partial \omega^2} \right|_{\omega=\omega_0}, \quad \delta = \frac{\Delta\omega_0}{\omega_0}, \quad g = \frac{2\pi n_2 \alpha}{\lambda}.$$

- Solitons can arise as solution of

$$i \frac{\partial \hat{E}}{\partial \xi} - \frac{k''}{2} \frac{\partial^2 \hat{E}}{\partial \tau^2} + g \frac{|\hat{E}|^2 \hat{E}}{\delta^2} = -\frac{i\gamma \hat{E}}{\delta^2}$$

if the real part of the nonlinear term is dominant,

$$g \frac{|\hat{E}|^2 \hat{E}}{\delta^2} > \frac{\gamma \hat{E}}{\delta^2} \quad \xrightarrow{g = \frac{2\pi n_2 \alpha}{\lambda} \approx \frac{\pi n_2}{\lambda}} \quad \frac{\pi n_2 |\hat{E}|^2}{\lambda} > \gamma$$

- the condition for existence of a soliton:

$$\frac{\pi n_2 |\hat{E}|^2}{\lambda} > \gamma$$

- example:

$$\left. \begin{array}{l}
 \lambda \approx 1500 \text{ nm} \\
 |\hat{E}| \approx 10^6 \text{ V/m} \\
 n_2 \approx 1.2 \times 10^{-22} \text{ m}^2/\text{V}^2
 \end{array} \right\} \gamma < 2 \times 10^{-4} \text{ m}^{-1}$$

↓

1.7 dB/km

- the normalized equation, with

$$\Gamma = \frac{\gamma\lambda}{\delta^2}$$

$$i \frac{\partial q}{\partial X} - \frac{1}{2} \frac{\partial^2 q}{\partial T^2} + |q|^2 q = -i\Gamma q$$

- if Γ is small enough, **perturbation techniques** can be used

$$q(T, X) = \eta(X) \operatorname{sech}(\eta(X)T) e^{-i\sigma(X)} + O(\Gamma)$$

$$\eta(X) = q_0 e^{-2\Gamma X}$$

$$\sigma(X) = \frac{q_0^2}{8\Gamma} (1 - e^{-4\Gamma X})$$

- The solution of the normalized soliton equation in fibers with loss predicts
 - the amplitude η of the soliton decreases as it propagates:

$$\eta(X) = q_0 e^{-2\Gamma X}$$

- the width σ of the soliton increases

$$\sigma(X) = \frac{q_0^2}{8\Gamma} (1 - e^{-4\Gamma X})$$

- **their product remains constant**

- Effects of the waveguide manifest as

$$i \left(\frac{\partial q}{\partial X} + \Gamma q \right) - \frac{1}{2} \frac{\partial^2 q}{\partial T^2} + |q|^2 q + i\delta \left(\beta_1 \frac{\partial^3 q}{\partial T^3} + \beta_2 \frac{\partial}{\partial T} (|q|^2 q) + \beta_3 q \frac{\partial}{\partial T} |q|^2 \right) = 0$$

higher order
linear dispersion

nonlinear dispersion of
the Kerr coefficient

nonlinear dissipation
due to Raman
processes
(imaginary!!!)

- Necessary condition for existence of a soliton

$$\tau_0 \sqrt{P_0} = 9.3 \times 10^{-2} \lambda^{3/2} \sqrt{|D|S}$$

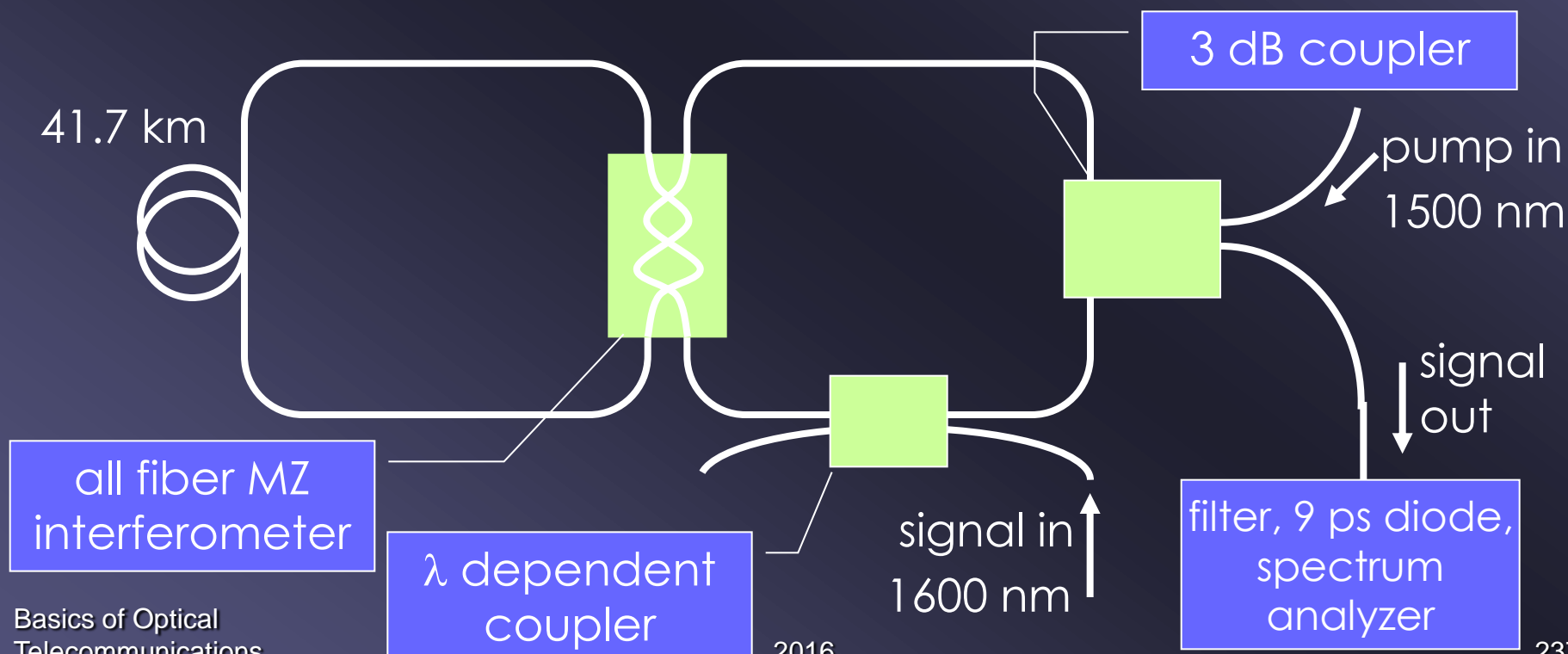
- τ_0 : pulse length [ps]
- P_0 : required pulse power [W]
- λ : wavelength [μm]
- D : dispersion [ps/(nm km)]
- S : cross-sectional area [μm^2]

e.g., $S=60 \mu\text{m}^2$, $\lambda=1.5 \mu\text{m}$, $|D|=10 \text{ ps}/(\text{nm km})$
 $\tau_0=10 \text{ ps}$, $P_0=180 \text{ mW}$

- Soliton generation needs
 - low loss fiber (<1 dB/km)
 - spectral width of the laser pulse be narrower than the inverse of the pulse length
 - Mollenauer & al. 1980, AT&T Bell Lab.
 - 700 m fiber, 10^{-6} cm² cross section
 - 7 ps pulse,
 - F²⁺ color center laser with Nd:YAG pump
 - 1.2 W soliton threshold

- For small loss the soliton propagates with the product of its pulse length and height being constant
- reshaping is needed for long-distance communication application
- reshaping methods:
 - induced Raman amplification – the loss compensated along the fiber
 - repeated Raman Amplifiers
 - Er doped amplifiers

- Experiment on the long distance transmission of a soliton by repeated Raman Amplification (Mollenauer & Smith, 1988)



- Erbium doped fiber amplifiers, periodically placed in the transmission line
 - distance of the amplifiers should be less than the soliton dispersion length
 - dispersion shifted fibers or filters for reshaping
- quantum noise arise
 - spontaneous emission noise
 - Gordon—House jitter

- The soliton based communication systems mostly use on/off or DPSK keying
- In soliton communication systems the timing jitters which originate from frequency fluctuation are held under control by narrow band optical filters
 - frequency guiding filter
 - e.g., a shallow Fabry-Perot etalon filter
(in non-soliton systems, these guiding filters destroy the signal, they are not used)

- It is possible to make the soliton “slide” in frequency
 - sliding frequency guiding filters
 - each consecutive narrow-band filter has slightly different center frequency
 - center frequency sliding rate: $f' = df/dz$
 - the solitons can follow the frequency shift
 - the noise can not follow the frequency sliding, it drops out

- Wavelength division multiplexing in soliton communication systems
 - solitons with different center frequency propagate with different group velocity
 - in collision of two solitons, they propagate together for a while
 - collision length:

$$L_{\text{coll}} = \frac{2\tau}{D\Delta\lambda}$$

- during the collision both solitons shifts in frequency (same magnitude, opposite sign)
- first part of the collision: the fast soliton's velocity increases, while the slow one becomes slower
- at the second part of the collision, the opposite effect takes place, symmetrically

- if during the collision the solitons reach an amplifier or a reshaper, the symmetry brakes
- the result is non-zero residual frequency shift can arise, unless

$$L_{\text{coll}} > 2L_{\text{amp}}$$

- if a collision of two solitons take place at the input of the transmission
- **half collision**
- it can be avoided by staggering the pulse positions of the WDM channels at the input.

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